Matrices

Question1

Let A be a square matrix of order 3 such that $\det(A) = -2$ and $\det(3\,\text{adj}(-6\,\text{adj}(3A))) = 2^{m+n} \cdot 3^{mn}, m > n$. Then $4m + 2n$ is equal to _____.

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Answer: 34

Solution:

$\text{As } A\,\text{adj}\,A = |A|I, \det(\lambda A) = \lambda^n \det A$

$\det(3\,\text{adj}(-6\,\text{adj}(3A))) = 3^3 \det(\text{adj}(-6\,\text{adj}(3A)))$

$= 3^3(-6\,\text{adj}(3A))^2$

$= 3^3(-6)^6|3A|^4$

$= 3^9 2^6 \cdot 3^{12} \cdot (-2)^4$

$= 3^{21} \cdot 2^{10}$

Now comparing with given condition

$2^{m+n}3^{mn} = 2^{10} \cdot 3^{21}$

$m + n = 10, mn = 21$

$\Rightarrow m = 7, n = 3 (m > n)$

$\therefore 4m + 2n = 28 + 6 = 34$

Question2

Let A be a 3×3 matrix such that $\mathrm{X}^{\mathrm{T}} \mathrm{AX}=\mathrm{O}$ for all nonzero 3×1 matrices $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$. If $\mathrm{A}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, \mathrm{A}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$, and $\det(\operatorname{adj}(2(\mathrm{A}+\mathrm{I})))=2^{\alpha} 3^{\beta} 5^{\gamma}, \alpha, \beta, \gamma \in N$, then $\alpha^2+\beta^2+\gamma^2$ is

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Answer: 44

Solution:

$X^T AX = 0$

$$(xyz)\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$(xyz)\begin{pmatrix} a_1x+a_2y+a_3z \\ b_1x+b_2y+b_3z \\ c_1x+c_2y+c_3z \end{pmatrix} = 0$$

$x\left(a_1x+a_2y+a_3z\right)+y\left(b_1x+b_2y+b_3z\right)$
$+z\left(c_1x+c_2y+c_3z\right)=0$
$a_1=0, b_2=0 c_3=0$
$a_2+b_1=0, a_3+c_1=0, b_3=c_2=0$

$A=$ skew symm matrix

$$A=\begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix}; \quad A=\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}=\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$

$$\Rightarrow A=\begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}=\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$

$x+y=1$
$-x+z=4$
$y+z=5$

$$\begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}=\begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$$

$2x+y=0 \qquad x=-1$
$-x+z=4 \qquad y=2$
$-y-2z=-8 \quad z=3$

$A=\begin{pmatrix}0 & -1 & 2\\ 1 & 0 & 3\\ -2 & -3 & 0\end{pmatrix}$

$2(\mathrm{A}+\mathrm{I})=\begin{pmatrix}2 & -2 & 4\\ 2 & 2 & 6\\ -2 & -6 & 2\end{pmatrix}$

$2(\mathrm{A}+\mathrm{I})=120 \Rightarrow \det|\operatorname{adi}(2(\mathrm{A}+\mathrm{I}))|$

$=120^2=2^6\cdot 3^2\cdot 5^2$

$\alpha=6, \beta=2, \gamma=2$

Question3

Let M denote the set of all real matrices of order 3×3 and let $\mathrm{S}=\{-3,-2,-1,1,2\}$. Let

$$\mathrm{S}_1=\left\{\mathrm{A}=[a_{\mathrm{ij}}]\in \mathrm{M}: \mathrm{A}=\mathrm{A}^{\mathrm{T}} \text{ and } a_{\mathrm{ij}}\in \mathrm{S}, \forall \mathrm{i},\mathrm{j}\right\},$$
$$\mathrm{S}_2=\left\{\mathrm{A}=[a_{\mathrm{ij}}]\in \mathrm{M}: \mathrm{A}=-\mathrm{A}^{\mathrm{T}} \text{ and } a_{\mathrm{ij}}\in \mathrm{S}, \forall \mathrm{i},\mathrm{j}\right\},$$
$$\mathrm{S}_3=\left\{\mathrm{A}=[a_{\mathrm{ij}}]\in \mathrm{M}: a_{11}+a_{22}+a_{33}=0 \text{ and } a_{\mathrm{ij}}\in \mathrm{S}, \forall \mathrm{i},\mathrm{j}\right\}.$$

If $n\left(S_1\cup S_2\cup S_3\right)=125\alpha$, then α equls _____.

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Answer: 1613

Solution:

$$\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$$

No. of elements in $\mathrm{S}_1: \mathrm{A}=\mathrm{A}^{\mathrm{T}} \Rightarrow 5^3\times 5^3$

No. of elements in $S_2: A=-A^T \Rightarrow 0$

since no zero in S_2

No. of elements in $\mathrm{S}_3 \Rightarrow$

$$\left.\begin{array}{c} a_{11}+a_{22}+a_{33}=0 \Rightarrow (1,2,-3) \Rightarrow 31 \\ \text{or} \\ (1,1,-2) \Rightarrow 3 \\ \text{or} \\ (-1,-1,2) \Rightarrow 3 \end{array}\right\} \Rightarrow 12 \times 5^6$$

$\mathrm{n}(\mathrm{S}_1 \cap \mathrm{S}_3) = 12 \times 5^3$

$\mathrm{n}(\mathrm{S}_1 \cup \mathrm{S}_2 \cup \mathrm{S}_3) = 5^6(1+12) - 12 \times 5^3$

$\Rightarrow 5^3 \times \left[13 \times 5^3 - 12\right] = 125\alpha$

$\alpha = 1613$

Question4

Let $S = \left\{m \in \mathbf{Z} : A^{m^2} + A^m = 3I - A^{-6}\right\}$, where $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$. Then $n(S)$ is equal to _____.

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Answer: 2

Solution:

$A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}, A^3 = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}, A^4 = \begin{bmatrix} 5 & -4 \\ 4 & -3 \end{bmatrix}$

and so on

$\mathrm{A}^6 = \begin{bmatrix} 7 & -6 \\ 6 & -5 \end{bmatrix}$

$\mathrm{A}^{\mathrm{m}} = \begin{bmatrix} \mathrm{m}+1 & -\mathrm{m} \\ \mathrm{m} & -\mathrm{m}+1 \end{bmatrix},$

$\mathrm{A}^{\mathrm{m}^2} = \begin{bmatrix} \mathrm{m}^2+1 & -\mathrm{m}^2 \\ \mathrm{m}^2 & -\left(\mathrm{m}^2-1\right) \end{bmatrix}$

$\mathrm{A}^{\mathrm{m}^2} + \mathrm{A}^{\mathrm{m}} = 3\mathrm{I} - \mathrm{A}^{-6}$

$\begin{bmatrix} \mathrm{m}^2+1 & -\mathrm{m}^2 \\ \mathrm{m}^2 & -\left(\mathrm{m}^2-1\right) \end{bmatrix} + \begin{bmatrix} \mathrm{m}+1 & -\mathrm{m} \\ \mathrm{m} & -\mathrm{m}+1 \end{bmatrix}$

$$= 3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}-5 & 6\\ -6 & 7\end{bmatrix}$$
$$= \begin{bmatrix}8 & -6\\ 6 & -4\end{bmatrix}$$
$$= \mathrm{m}^2 + 1 + \mathrm{m} + 1 = 8$$
$$= \mathrm{m}^2 + \mathrm{m} - 6 = 0 \Rightarrow \ \mathrm{m} = -3, 2$$
$$\mathrm{n}(\ \mathrm{s}) = 2$$

Question5

Let I be the identity matrix of order 3×3 and for the matrix $A = \begin{bmatrix}\lambda & 2 & 3\\ 4 & 5 & 6\\ 7 & -1 & 2\end{bmatrix}, |A| = -1$. Let B be the inverse of the matrix $\mathrm{adj}\left(\mathrm{Aadj}\left(A^2\right)\right)$. Then $|(\lambda \mathrm{B} + \mathrm{I})|$ is equal to_____

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Answer: 38

Solution:

$$B = \left[\mathrm{adj}\left(A\,\mathrm{adj}\left(A^2\right)\right)\right]^{-1}$$
$$\mathrm{Adj}\left(A^2\right) = (\mathrm{adj}\,A)^2 \Rightarrow A\,\mathrm{adj}\left(A^2\right) = A\,\mathrm{adj}(A)\cdot(\mathrm{adj}\,A)$$
$$= A\left(|A|A^{-1}\right)^2 = |A|^2\left(A^{-1}\right) = A^{-1}$$
$$\Rightarrow B = \left(\mathrm{adj}\left(A^{-1}\right)\right)^{-1} = \left(\left|\left(A^{-1}\right)\right|A\right)^{-1} = \frac{A^{-1}}{-1} = -A^{-1}$$
$$\Rightarrow B = -A^{-1}$$

$$|A| = -1 = \begin{vmatrix}\lambda & 2 & 3\\ 4 & 5 & 6\\ 7 & -1 & 2\end{vmatrix} = -1 \Rightarrow \lambda = 3$$

$$|3B + I| = \left|I - 3A^{-1}\right| = \frac{|A|\left|I - 3A^{-1}\right|}{|A|} = \frac{|A - 3I|}{|A|}$$

$$= \frac{|A - 3I|}{-1} = \frac{\begin{vmatrix}0 & 2 & 3\\ 4 & 2 & 6\\ 7 & -1 & -1\end{vmatrix}}{-1} = 38$$
$$\Rightarrow |3B + I| = 38$$

Question6

Let $A=\begin{bmatrix}\cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta\end{bmatrix}$. If for some $\theta \in (0,\pi)$, $A^2=A^T$, then the sum of the diagonal elements of the matrix $(\mathrm{A}+\mathrm{I})^3+(\mathrm{A}-\mathrm{I})^3-6\ \mathrm{A}$ is equal to _____ .

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Answer: 6

Solution:

Note that A is orthogonal:

$AA^T=A^TA=I$ and $A^T=A^{-1}$

Given $A^2=A^T$, then:

$A^3=I$
$\mathrm{Tr}(A+I)^3+(A-l)^3-6A=\mathrm{Tr}\left(2A^3+6A-6A\right)$
$=\mathrm{Tr}\left(2A^3\right)=\mathrm{Tr}(2I)$
$\left(\text{ Using } (A+l)^3+(A-I)^3=2A^3+6A \text{ and } 2A^3=2I\right)=$
6

Question7

The number of singular matrices of order 2 , whose elements are from the set $\{2,3,6,9\}$, is _____.

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Answer: 36

Solution:

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

for A to be singular matrix

$ad = bc$

Case 1: exactly 1 number is used $\Rightarrow {}^4C_1$ ways

Case 2 : exactly 2 numbers is used $\Rightarrow {}^4C_2$ ways

Case 3 : exactly 3 numbers used $\Rightarrow$ none will be singular.

Case 4: exactly 4 numbers is used

$\Rightarrow ab = cd \Rightarrow 2 \times 9 = 3 \times 6$
$\Rightarrow {}^4C_1 \times 2! = 8$ matrix .

$\therefore$ Total ways $\Rightarrow 4 + 6 \times 4 + 8 = 36$ matrices.

Question8

For a 3×3 matrix M, let trace (M) denote the sum of all the diagonal elements of M. Let A be a 3×3 matrix such that $|A| = \frac{1}{2}$ and trace $(A) = 3$. If $B = \text{adj}(\text{adj}(2A))$, then the value of $|B|+$ trace (B) equals :

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Options:

A. 56

B. 132

C. 174

D. 280

Answer: D

Solution:

$B = \text{adj}(\text{adj}(2A)) = \det(2A) \cdot (2A)$

Since A is a 3×3 matrix with

$\det(A) = \frac{1}{2}$,

the determinant of $2A$ is computed as

$\det(2A) = 2^3 \det(A) = 8 \cdot \frac{1}{2} = 4.$

Thus,

$B = 4 \cdot (2A) = 8A.$

Now, compute the determinant and the trace of B:

Determinant of B:

$\det(B) = \det(8A) = 8^3 \det(A) = 512 \cdot \frac{1}{2} = 256.$

Trace of B:

$\text{trace}(B) = \text{trace}(8A) = 8 \cdot \text{trace}(A) = 8 \cdot 3 = 24.$

Finally, adding these results:

$\det(B) + \text{trace}(B) = 256 + 24 = 280.$

Question9

If the system of linear equations :

$$
\begin{aligned}
& x+y+2 z=6 \\
& 2 x+3 y+\mathrm{az}=\mathrm{a}+1 \\
& -x-3 y+\mathrm{bz}=2 \mathrm{~b}
\end{aligned}
$$

where $a, b \in \mathbf{R}$, has infinitely many solutions, then $7a + 3b$ is equal to :

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Options:

A. 12

B. 9

C. 22

D. 16

Answer: D

Solution:

We begin with the system:

$$\begin{aligned} x + y + 2z &= 6, \\ 2x + 3y + az &= a + 1, \\ -x - 3y + bz &= 2b. \end{aligned}$$

Step 1. Solve the first equation for x:

$x = 6 - y - 2z.$

Step 2. Substitute $x = 6 - y - 2z$ into the second equation:

$2(6 - y - 2z) + 3y + az = a + 1.$

Expanding and simplifying:

$12 - 2y - 4z + 3y + az = a + 1 \quad \Longrightarrow \quad y + (a - 4)z = a - 11.$

Call this **Equation (I)**.

Step 3. Substitute $x = 6 - y - 2z$ into the third equation:

$-(6 - y - 2z) - 3y + bz = 2b.$

Expanding and simplifying:

$-6 + y + 2z - 3y + bz = 2b \quad \Longrightarrow \quad -2y + (b + 2)z = 2b + 6.$

Call this **Equation (II)**.

Step 4. For the system to have infinitely many solutions, the two equations in y and z must be dependent—that is, one must be a constant multiple of the other. Assume there exists a constant k such that

$-2 = k \cdot 1 \quad \Longrightarrow \quad k = -2.$

Apply this to the coefficient of z and the constant term.

For the z-coefficient in Equations (I) and (II):

$b + 2 = k(a - 4) = -2(a - 4) = -2a + 8.$

Thus,

$b = -2a + 6.$

For the constant term:

$2b + 6 = k(a - 11) = -2(a - 11) = -2a + 22.$

Substitute $b = -2a + 6$ into this equation:

$2(-2a + 6) + 6 = -2a + 22 \quad \Longrightarrow \quad -4a + 12 + 6 = -2a + 22.$

Simplify:

$-4a + 18 = -2a + 22.$

Solve for a:

$-4a + 18 + 4a = -2a + 22 + 4a \quad \Longrightarrow \quad 18 = 2a + 22,$

$2a = 18 - 22 = -4 \quad \Longrightarrow \quad a = -2.$

Substitute $a = -2$ into $b = -2a + 6$:

$b = -2(-2) + 6 = 4 + 6 = 10.$

Step 5. We now compute

$7a + 3b = 7(-2) + 3(10) = -14 + 30 = 16.$

Thus, the value of $7a + 3b$ is

$\boxed{16}$.

Question10

If A, B, and $\left(\operatorname{adj}\left(\mathrm{A}^{-1}\right)+\operatorname{adj}\left(\mathrm{B}^{-1}\right)\right)$ are non-singular matrices of same order, then the inverse of $A\left(\operatorname{adj}\left(A^{-1}\right)+\operatorname{adj}\left(B^{-1}\right)\right)^{-1} B$, is equal to

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Options:

A. $\frac{AB^{-1}}{|A|}+\frac{BA^{-1}}{|B|}$

B. $\operatorname{adj}\left(\mathrm{B}^{-1}\right)+\operatorname{adj}\left(\mathrm{A}^{-1}\right)$

C. $\mathrm{AB}^{-1}+\mathrm{A}^{-1} \mathrm{B}$

D. $\frac{1}{|AB|}(\operatorname{adj}(B)+\operatorname{adj}(A))$

Answer: D

Solution:

$$\left[\mathrm{A}\left(\operatorname{adj}\left(\mathrm{A}^{-1}\right)+\operatorname{adj}\left(\mathrm{B}^{-1}\right)\right)^{-1} \cdot \mathrm{B}\right]^{-1}$$

$$\mathrm{B}^{-1} \cdot\left(\operatorname{adj}\left(\mathrm{A}^{-1}\right)+\operatorname{adj}\left(\mathrm{B}^{-1}\right)\right) \cdot \mathrm{A}^{-1}$$

$$\mathrm{B}^{-1} \operatorname{adj}\left(\mathrm{A}^{-1}\right) \mathrm{A}^{-1}+\mathrm{B}^{-1}\left(\operatorname{adj}\left(\mathrm{B}^{-1}\right)\right) \cdot \mathrm{A}^{-1}$$

$$\mathrm{B}^{-1}\left|\mathrm{A}^{-1}\right| \mathrm{I}+\left|\mathrm{B}^{-1}\right| \mathrm{I} \mathrm{A}^{-1}$$

$$\frac{\mathrm{B}^{-1}}{|\mathrm{A}|}+\frac{\mathrm{A}^{-1}}{|\mathrm{B}|}$$

$$\Rightarrow \frac{\mathrm{adjB}}{|\mathrm{B}||\mathrm{A}|}+\frac{\mathrm{adj}}{|\mathrm{A}||\mathrm{B}|}$$

$$=\frac{1}{|\mathrm{A}||\mathrm{B}|}(\mathrm{adjB}+\mathrm{adjA})$$

Question11

If the system of equations

$$(\lambda-1)x+(\lambda-4)y+\lambda z=5$$
$$\lambda x+(\lambda-1)y+(\lambda-4)z=7$$
$$(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$$

has infinitely many solutions, then $\lambda^2+\lambda$ is equal to

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Options:

A. 20

B. 10

C. 6

D. 12

Answer: D

Solution:

$(\lambda-1)x+(\lambda-4)y+\lambda z=5$
$\lambda x+(\lambda-1)y+(\lambda-4)z=7$
$(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$
For infinitely many solutions

$$\mathrm{D}=\begin{vmatrix}\lambda-1 & \lambda-4 & \lambda \\ \lambda & \lambda-1 & \lambda-4 \\ \lambda+1 & \lambda+2 & -(\lambda+2)\end{vmatrix}=0$$

$(\lambda-3)(2\lambda+1)=0$

$$\mathrm{D_x}=\begin{vmatrix}5 & \lambda-4 & \lambda \\ 7 & \lambda-1 & \lambda-4 \\ 9 & \lambda+2 & -(\lambda+2)\end{vmatrix}=0$$

$2(3-\lambda)(23-2\lambda)=0$
$\lambda=3$
$\therefore \lambda^2+\lambda=9+3=12$

Question12

Let $A=[a_{ij}]$ be a 3×3 matrix such that $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}, A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$ and $A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, then a_{23} equals :

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Options:

A. 2

B. -1

C. 1

D. 0

Answer: B

Solution:

$$\text{Let } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} a_{22} = 0; a_{12} = 0 \\ a_{32} = 1 \end{matrix}$$

$$A \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{matrix} \\ \Rightarrow \\ \\ \end{matrix} \begin{matrix} 4a_{11} + a_{12} + 3a_{13} = 0 \\ 4a_{21} + a_{22} + 3a_{23} = 1 \Rightarrow 4a_{21} + 3a_{23} = 1 \\ 4a_{31} + a_{32} + 3a_{33} = 0 \end{matrix}$$

$$A \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \\ \Rightarrow \\ \\ \end{matrix} \begin{matrix} 2a_{11} + a_{12} + 2a_{13} = 1 \\ 2a_{21} + a_{22} + 2a_{23} = 0 \Rightarrow a_{21} + a_{23} = 0 \\ 2a_{31} + a_{32} + 2a_{33} = 0 \end{matrix}$$

$$-4a_{23} + 3a_{23} = 1 \Rightarrow a_{23} = -1$$

Question13

The system of equations

$$x + y + z = 6,$$
$$x + 2y + 5z = 9,$$
$$x + 5y + \lambda z = \mu,$$

has no solution if

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Options:

A. $\lambda = 17, \mu = 18$

B. $\lambda = 17, \mu \neq 18$

C. $\lambda = 15, \mu \neq 17$

D. $\lambda \neq 17, \mu \neq 18$

Answer: B

Solution:

$$\mathrm{D}=\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 5\\ 1 & 5 & \lambda\end{vmatrix}=0$$

$\lambda=17$

$$D_z=\begin{vmatrix}1 & 1 & 6\\ 1 & 2 & 9\\ 1 & 5 & \mu\end{vmatrix}\neq 0$$

$\mu\neq 18$

Question14

If the system of equations

$$2x-y+z=4$$
$$5x+\lambda y+3z=12$$
$$100x-47y+\mu z=212$$

has infinitely many solutions, then $\mu-2\lambda$ is equal to

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Options:

A. 56

B. 59

C. 57

D. 55

Answer: C

Solution:

$$\Delta = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ 5 & \lambda & 3 \\ 100 & -47 & \mu \end{vmatrix} = 0$$

$2(\lambda\mu + 141) + (5\mu - 300) - 235 - 100\lambda = 0 \ldots (1)$

$$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 4 \\ 5 & \lambda & 12 \\ 100 & -47 & 212 \end{vmatrix} = 0$$

$6\lambda = -12 \Rightarrow \lambda = -2$

Put $\lambda = 2$ in (1)

$2(-2\mu + 141) + 5\mu - 300 - 235 + 200 = 0$

$\mu = 53$

$\therefore 57$

Question15

If the system of equations

$$x + 2y - 3z = 2$$
$$2x + \lambda y + 5z = 5$$
$$14x + 3y + \mu z = 33$$

has infinitely many solutions, then $\lambda + \mu$ is equal to :

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Options:

A. 13

B. 10

C. 12

D. 11

Answer: C

Solution:

$$\mathrm{D}=\begin{vmatrix}1 & 2 & -3\\ 2 & \lambda & 5\\ 14 & 3 & \mu\end{vmatrix}=0, \lambda\mu+42\lambda-4\mu+107=0$$

$\mathrm{D}_1=2\lambda\mu+99\lambda-10\mu+255$

$\mathrm{D}_2=13-\mu$

$D_3=5\lambda+5$

$D_2=0\Rightarrow\mu=13 \& D_3=0\Rightarrow\lambda=-1$

check & verify for these values $\mathrm{D} \& \mathrm{D}_2=0$

Question16

For some a, b, let

$$f(x)=\begin{vmatrix}\mathrm{a}+\frac{\sin x}{x} & 1 & \mathrm{b}\\ \mathrm{a} & 1+\frac{\sin x}{x} & \mathrm{b}\\ \mathrm{a} & 1 & \mathrm{b}+\frac{\sin x}{x}\end{vmatrix}, x\neq 0, \lim_{x\to 0} f(x)=\lambda+\mu\mathrm{a}+\nu\mathrm{b}.$$

Then $(\lambda+\mu+v)^2$ is equal to :

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Options:

A. 25

B. 16

C. 9

D. 36

Answer: B

Solution:

$$\lim_{x\to 0} f(x)=\begin{vmatrix}a+1 & 1 & b\\ a & 1+1 & b\\ a & 1 & b+1\end{vmatrix}$$

$=(a+1)(2(b+1)-b)+1(ab-a(b+1))+ba$

$=(a+1)(b+2)-a+ab$

$=b+a+2=\lambda+\mu a+vb$

$\lambda=2, \mu=1, v=1\Rightarrow(\lambda+\mu+v)^2=16$

Question17

Let $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix}$ **and** $P = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \theta > 0$**. If** $B = PAP^{\top}, C = P^{\top}B^{10}P$ **and the sum of the diagonal elements of** C **is** $\frac{m}{n}$**, where** $\gcd(m, n) = 1$**, then** $m + n$ **is :**

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Options:

A.

127

B.

2049

C.

258

D.

65

Answer: D

Solution:

$P = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$\because P^{T}P = I$

$B = PAPT$

Pre multiply by P^{T} (Given)

$P^{T}\,B = P^{T}PAP^{T} = AP^{T}$

Now post multiply by P

$P^{T}BP = AP^{T}P = A$

$$\text{So } A^2 = \underbrace{P^{T}BP\ P^{T}}_{I}\ BP$$

$\mathrm{A}^2 = \mathrm{P}^{\mathrm{T}}\,\mathrm{B}^2\mathrm{P}$

Similarly $A^{10} = P^T B^{10} P = C$

$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix}$ (Given)

$\Rightarrow A^2 = \begin{bmatrix} \frac{1}{2} & -\sqrt{2}-2 \\ 0 & 1 \end{bmatrix}$

Similarly check A^3 and so on since $\mathrm{C} = \mathrm{A}^{10}$

$\Rightarrow$ Sum of diagonal elements of C is $\left(\frac{1}{\sqrt{2}}\right)^{10} + 1$

$= \dfrac{1}{32} + 1 = \dfrac{33}{32} = \dfrac{\mathrm{m}}{\mathrm{n}}$

$\mathrm{g\,cd}(\,\mathrm{m}, \mathrm{n}) = 1$ (Given)

$\Rightarrow \mathrm{m} + \mathrm{n} = 65$

Question18

Let M and m respectively be the maximum and the minimum values of

$$f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ \sin^2 x & 1+\cos^2 x & 4\sin 4x \\ \sin^2 x & \cos^2 x & 1+4\sin 4x \end{vmatrix}, x \in R$$

Then $M^4 - m^4$ is equal to :

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Options:

A.

1280

B.

1040

C.

1215

D.

1295

Answer: A

Solution:

$$\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ \sin^2 x & 1+\cos^2 x & 4\sin 4x \\ \sin^2 x & \cos^2 x & 1+4\sin 4x \end{vmatrix}, x \in R$$

$R_2 \to R_2 - R_1 \& R_3 \to R_3 - R_1$

$$f(x)\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Expand about R_1, we get

$f(x) = 2 + 4\sin 4x$

$\therefore M =$ max value of $f(x) = 6$

$\mathrm{m} =$ min value of $\mathrm{f(x)} = -2$

$\therefore \mathrm{M}^4 - \mathrm{m}^4 = 1280$

Question19

Let $A = [a_{ij}] = \begin{bmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{bmatrix}$. If A_{ij} is the cofactor of a_{ij}, $C_{ij} = \sum_{k=1}^{2} a_{ik}A_{jk}, 1 \le i, j \le 2$, and $C = [C_{ij}]$, then $8|C|$ is equal to :

JEE Main 2025 (Online) 29th January Morning Shift

Options:

A.

288

B.

262

C.

222

D.

242

Answer: D

Solution:

To solve the problem, we need to determine the determinant of matrix A:

$$A = \begin{bmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{bmatrix}$$

The determinant of A, denoted as $|A|$, is calculated as:

$$|A| = (\log_5 128)(\log_4 25) - (\log_4 5)(\log_5 8)$$

Evaluating each component:

$\log_5 128$ can be simplified using change of base formula:

$$\log_5 128 = \frac{\log_{10} 128}{\log_{10} 5}$$

$\log_4 5$ using change of base:

$$\log_4 5 = \frac{\log_{10} 5}{\log_{10} 4}$$

$\log_5 8$:

$$\log_5 8 = \frac{\log_{10} 8}{\log_{10} 5}$$

$\log_4 25$:

$$\log_4 25 = \frac{\log_{10} 25}{\log_{10} 4}$$

Now, substitute these values into $|A|$:

$$|A| = \left(\frac{\log_{10} 128}{\log_{10} 5}\right)\left(\frac{\log_{10} 25}{\log_{10} 4}\right) - \left(\frac{\log_{10} 5}{\log_{10} 4}\right)\left(\frac{\log_{10} 8}{\log_{10} 5}\right)$$

Next, we find the cofactors of matrix A:

$$A_{11} = \log_4 25$$

$$A_{12} = -\log_5 8$$

$$A_{21} = -\log_4 5$$

$$A_{22} = \log_5 128$$

Then, calculate matrix C whose elements are given by $C_{ij} = \sum_{k=1}^{2} a_{ik} A_{jk}$:

$C_{11} = a_{11}A_{11} + a_{12}A_{12} = |A| = \frac{11}{2}$

$C_{12} = a_{11}A_{21} + a_{12}A_{22} = 0$

$C_{21} = a_{21}A_{11} + a_{22}A_{12} = 0$

$C_{22} = a_{21}A_{21} + a_{22}A_{22} = |A| = \frac{11}{2}$

Thus, matrix C is:

$$C = \begin{bmatrix} \frac{11}{2} & 0 \\ 0 & \frac{11}{2} \end{bmatrix}$$

To find $|C|$, we compute:

$|C| = \left(\frac{11}{2}\right)\left(\frac{11}{2}\right) = \frac{121}{4}$

Finally, calculate $8|C|$:

$8|C| = 8 \times \frac{121}{4} = 242$

Question20

Let $\mathrm{A} = [a_{ij}]$ be a matrix of order 3×3, with $a_{ij} = (\sqrt{2})^{i+j}$. If the sum of all the elements in the third row of A^2 is $\alpha + \beta\sqrt{2}, \alpha, \beta \in \mathbf{Z}$, then $\alpha + \beta$ is equal to :

JEE Main 2025 (Online) 29th January Evening Shift

Options:

A.

210

B.

280

C.

224

D.

168

Answer: C

Solution:

$$A=\begin{bmatrix}(\sqrt{2})^2 & (\sqrt{2})^3 & (\sqrt{2})^4\\ (\sqrt{2})^3 & (\sqrt{2})^4 & (\sqrt{2})^5\\ (\sqrt{2})^4 & (\sqrt{2})^5 & (\sqrt{2})^6\end{bmatrix}$$

$$A=\begin{bmatrix}2 & 2\sqrt{2} & 4\\ 2\sqrt{2} & 4 & 4\sqrt{2}\\ 4 & 4\sqrt{2} & 8\end{bmatrix}$$

$$A^2=2^2\begin{bmatrix}1 & \sqrt{2} & 2\\ \sqrt{2} & 2 & 2\sqrt{2}\\ 2 & 2\sqrt{2} & 4\end{bmatrix}\begin{bmatrix}1 & \sqrt{2} & 2\\ \sqrt{2} & 2 & 2\sqrt{2}\\ 2 & 2\sqrt{2} & 4\end{bmatrix}$$

$$=4\begin{bmatrix}- & - & -\\ - & - & -\\ (2+4+8) & (2\sqrt{2}+4\sqrt{2}+8\sqrt{2}) & (4+8+16)\end{bmatrix}$$

Sum of elements of 3^{rd} row $=4(14+14\sqrt{2}+28)$

$=4(42+14\sqrt{2})$

$=168+56\sqrt{2}$

$\alpha+\beta\sqrt{2}$

$\therefore\quad \alpha+\beta=168+56=224$

Question21

Let $\alpha,\beta\ (\alpha\neq\beta)$ be the values of m, for which the equations $x+y+z=1, x+2y+4z=m$ and $x+4y+10z=m^2$ have infinitely many solutions. Then the value of $\sum_{n=1}^{10}(n^{\alpha}+n^{\beta})$ is equal to :

JEE Main 2025 (Online) 29th January Evening Shift

Options:

A.

3410

B.

560

C.

3080

D.

440

Answer: D

Solution:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 1(20-16) - 1(10-4) + 1(4-2)$$

$$= 4 - 6 + 2 = 0$$

For infinite solutions

$\Delta_x = \Delta_y = \Delta_z = 0$

$m^2 - 3x + 2 = 0$

$m = 1, 2$

$\alpha = 1, \beta = 2$

$$\therefore \sum_{n=1}^{10} \left(n^{\alpha} + n^{\beta}\right) = \sum_{n=1}^{10} n^1 + \sum_{n=1}^{10} n^2$$

$$= \frac{10(11)}{2} + \frac{10(11)(21)}{6}$$

$$= 55 + 385$$

$$= 440$$

Question22

Let $A = [a_{ij}]$ be a 2×2 matrix such that $a_{ij} \in \{0, 1\}$ for all i and j. Let the random variable X denote the possible values of the determinant of the matrix A. Then, the variance of X is:

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Options:

A.

$\frac{5}{8}$

B.

$\frac{1}{4}$

C.

$\frac{3}{4}$

D.

$\frac{3}{8}$

Answer: D

Solution:

$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$
$= a_{11}a_{22} - a_{21}a_{12}$
$= \{-1, 0, 1\}$

x	P_i	P_iX_i	$P_1{X_i}^2$
-1	$\frac{3}{16}$	$-\frac{3}{16}$	$\frac{3}{16}$
0	$\frac{10}{16}$	0	0
1	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$
		$\sum P_iX_i = 0$	$\sum P_i{X_i}^2 = \frac{3}{8}$

$\therefore \text{var(x)} = \sum P_iX_i^2 - \left(\sum P_iX_i\right)^2$
$= \frac{3}{8} - 0 = \frac{3}{8}$

Question23

If the system of linear equations

$$3x + y + \beta z = 3$$
$$2x + \alpha y - z = -3$$
$$x + 2y + z = 4$$

has infinitely many solutions, then the value of $22\beta - 9\alpha$ is :

JEE Main 2025 (Online) 2nd April Morning Shift

Options:

A. 31

B. 37

C. 43

D. 49

Answer: A

Solution:

$3x + y + \beta z = 3$
$2x + \alpha y - z = -3$
$x + 2y + z = 4$
has infinite solution
$\Rightarrow \Delta = 0, \Delta_1 = \Delta_2 = \Delta_3$

$$\Delta = 0 \Rightarrow \begin{vmatrix} 3 & 1 & \beta \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\Delta_2 = 0 \Rightarrow \begin{vmatrix} 3 & 3 & \beta \\ 2 & -3 & -1 \\ 1 & 4 & 1 \end{vmatrix} = 0$$

$\Rightarrow 3(-3+4) - 3(2+1) + \beta(8+3) = 0$
$\Rightarrow 3 - 9 + 11\beta = 0$
$\Rightarrow \quad \beta = \dfrac{6}{11}$

$$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 3 & 1 & 3 \\ 2 & \alpha & -3 \\ 1 & 2 & 4 \end{vmatrix} = 0$$

$\Rightarrow \quad 3(4\alpha + 6) - 1(8+3) + 3(4-\alpha) = 0$
$12\alpha + 18 - 11 + 12 - 3\alpha = 0$
$9\alpha = -19$
$\alpha = \dfrac{-19}{9}$

$\therefore \quad 22\beta - 9\alpha = 31$

Question24

Let $a \in R$ and A be a matrix of order 3×3 such that $\det(A) = -4$ and $A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$, where I is the identity matrix of order 3×3. If $\det((a+1)\,\mathrm{adj}((a-1)A))$ is $2^{\mathrm{m}}3^{\mathrm{n}}, \mathrm{m}, \mathrm{n} \in \{0, 1, 2, \ldots, 20\}$, then $\mathrm{m} + \mathrm{n}$ is equal to :

JEE Main 2025 (Online) 2nd April Morning Shift

Options:

A. 14

B. 17

C. 15

D. 16

Answer: D

Solution:

To solve for the value of $m+n$, we first establish the matrix A and determine the value of a. The condition given is:

$$A+I=\begin{bmatrix}1 & a & 1\\ 2 & 1 & 0\\ a & 1 & 2\end{bmatrix}$$

Since I is the identity matrix of size 3×3, we have:

$$A=\begin{bmatrix}1 & a & 1\\ 2 & 1 & 0\\ a & 1 & 2\end{bmatrix}-I=\begin{bmatrix}0 & a & 1\\ 2 & 0 & 0\\ a & 1 & 1\end{bmatrix}$$

We know $\det(A)=-4$. Calculating the determinant:

$\det(A)=0(0\times 1-0\times 1)-a(2\times 1-0\times a)+1(2\times 1-0\times 1)=-2a+2$

Setting $-2a+2=-4$, we solve for a:

$-2a+2=-4$

$-2a=-6$

$a=3$

With $a=3$, the problem is to find $\det((a+1)\,\text{adj}((a-1)A))$. With $a=3$:

Calculate $(a+1)\,\text{adj}((a-1)A)$:

$=4\,\text{adj}(3A)$

Calculate the determinant:

$\det(4\,\text{adj}(3A))=4^3\times\det(\text{adj}(3A))$

Using the property $\det(\text{adj}(A))=\det(A)^{n-1}$ for a $n\times n$ matrix:

$\text{adj}(3A)=(3A)^{2-1}=(3\times\det(A))^2$

$= (3^3 \times (-4))^2$

Therefore, calculate $\det(3A)^2$:

$\det((3A)^2) = (3^3 \times (-4))^2 = 3^6 \times 4^2$

Substitute back:

$4^3 \times \det((3A)^2) = 4^3 \times 3^6 \times 4^2 = 2^6 \times 3^6 \times 2^4$

Simplifying gives:

$= 2^{10} \times 3^6$

Finally, we have powers $m = 10$ and $n = 6$. Therefore:

$m + n = 10 + 6 = 16$

Question25

Let $\mathrm{A} = \begin{bmatrix} \alpha & -1 \\ 6 & \beta \end{bmatrix}, \alpha > 0$**, such that** $\det(\mathrm{A}) = 0$ **and** $\alpha + \beta = 1$**. If I denotes** 2×2 **identity matrix, then the matrix** $(I + A)^8$ **is :**

JEE Main 2025 (Online) 2nd April Morning Shift

Options:

A. $\begin{bmatrix} 257 & -64 \\ 514 & -127 \end{bmatrix}$

B. $\begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

C. $\begin{bmatrix} 1025 & -511 \\ 2024 & -1024 \end{bmatrix}$

D. $\begin{bmatrix} 4 & -1 \\ 6 & -1 \end{bmatrix}$

Answer: B

Solution:

Let $|A| = 0 \Rightarrow \alpha\beta - (-6) = 0 \Rightarrow \alpha\beta = -6$ and $\alpha + \beta = 1 \Rightarrow \alpha\beta$ are roots of the equation

$x^2 - x - 6 = 0 \Rightarrow x = 3, -2$. Since $\alpha > 0$

$\Rightarrow \quad \alpha = 3, \beta = -2$

$\Rightarrow \quad A = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} \Rightarrow I + A = \begin{bmatrix} 4 & -1 \\ 6 & -1 \end{bmatrix}$

$(I + A)^2 = \begin{bmatrix} 10 & -3 \\ 18 & -5 \end{bmatrix} \Rightarrow (I + A)^4 = \begin{bmatrix} 46 & -15 \\ 90 & -29 \end{bmatrix}$

$\Rightarrow \quad (I + A)^8 = \begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

Question26

Let A be a 3×3 real matrix such that $A^2(A - 2I) - 4(A - I) = O$, where I and O are the identity and null matrices, respectively. If $A^5 = \alpha A^2 + \beta A + \gamma I$, where α, β, and γ are real constants, then $\alpha + \beta + \gamma$ is equal to :

JEE Main 2025 (Online) 2nd April Evening Shift

Options:

A. 76

B. 12

C. 4

D. 20

Answer: B

Solution:

$A^2(A - 2I) - 4(A - I) = 0$
$A^3 - 2A^2 - 4A + 4I = 0$

Multiply by A

$A^4 = 2A^3 + 4A^2 - 4A$
$A^4 = 2\left(2A^2 + 4A - 4I\right) + 4A^2 - 4A$
$A^4 = 8A^2 + 4A - 8I$

Multiply again by A

$\Rightarrow A^5 = 8A^3 + 4A^2 - 8A$
$\Rightarrow A^5 = 8\left(2A^2 + 4A - 4I\right) + 4A^2 - 8A$
$\Rightarrow A^5 = 20A^2 + 24A - 32I$

Comparing with $A^5 = \alpha A^2 + \beta A + \gamma I$

$\alpha = 20, \beta = 24, \gamma = -32$
$\therefore \alpha + \beta + \gamma = 20 + 24 - 32$
$= 44 - 32$
$= 12$

Question27

If the system of equations

$$2x + \lambda y + 3z = 5$$
$$3x + 2y - z = 7$$
$$4x + 5y + \mu z = 9$$

has infinitely many solutions, then $\left(\lambda^2 + \mu^2\right)$ is equal to :

JEE Main 2025 (Online) 2nd April Evening Shift

Options:

A. 30

B. 26

C. 22

D. 18

Answer: B

Solution:

$2x + \lambda y + 3z = 5$
$3x + 2y - z = 7$
$4x + 5y + \mu z = 9$
For infinite solutions $\Rightarrow \Delta = 0 = \Delta_1 = \Delta_2 = \Delta_3$

$$\Delta = \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$$

$$\Rightarrow -4\lambda - 3\lambda\mu + 4\mu + 31 = 0$$

$$\Delta_1 = \begin{vmatrix} 5 & \lambda & 3 \\ 7 & 2 & -1 \\ 9 & 5 & \mu \end{vmatrix} = 0 \Rightarrow -9\lambda - 7\lambda\mu + 10\mu + 76 = 0$$

$$\Delta_2 = \begin{vmatrix} 2 & 3 & 5 \\ 3 & -1 & 7 \\ 4 & \mu & 9 \end{vmatrix} = 0 \Rightarrow \mu + 5 = 0 \Rightarrow \mu = -5$$

$$\Delta_3 = \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0 \Rightarrow \lambda + 1 = 0 \Rightarrow \lambda = -1$$

$\therefore$ For infinite solution $\mu = -5$ and $\lambda = -1$

Now $\mu^2 + \lambda^2 = 25 + 1$

$= 26$

Question28

Let A be a matrix of order 3×3 and $|A| = 5$. If $|2\,\text{adj}(3A\,\text{adj}(2A))| = 2^{\alpha} \cdot 3^{\beta} \cdot 5^{\gamma}, \alpha, \beta, \gamma \in N$, then $\alpha + \beta + \gamma$ is equal to

JEE Main 2025 (Online) 3rd April Morning Shift

Options:

A. 26

B. 27

C. 25

D. 28

Answer: B

Solution:

To find the expression $|2\,\text{adj}(3A\,\text{adj}(2A))|$, we break it down as follows:

Recognize that:

$|2\,\text{adj}(3A\,\text{adj}(2A))| = 2^3|3A(\text{adj}(2A))|^2$

Apply properties of determinants:

$= 2^3(3^3)^2|A|^2|\text{adj}(2A)|^2$

Further simplify using $|\text{adj}(B)| = |B|^{n-1}$ for a 3×3 matrix:

$= 2^3 \cdot 3^6 \cdot 5^2 \cdot (|2A|^2)^2$

Simplify $|2A|$:

$= 2^3 \cdot 3^6 \cdot 5^2 \cdot (2^3)^4 \cdot |A|^4$

Continue to simplify:

$= 2^3 \cdot 3^6 \cdot 5^2 \cdot (2^3)^4 \cdot 5^4$

Expand and combine powers:

$= 2^{15} \cdot 3^6 \cdot 5^6$

Therefore, $\alpha = 15$, $\beta = 6$, and $\gamma = 6$. So, $\alpha + \beta + \gamma = 15 + 6 + 6 = 27$.

Question29

Let the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ satisfy $A^n = A^{n-2} + A^2 - I$ for $n \geqslant 3$.
Then the sum of all the elements of A^{50} is :

JEE Main 2025 (Online) 4th April Evening Shift

Options:

A. 44

B. 39

C. 52

D. 53

Answer: D

Solution:

$$A=\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}$$

$$A^3=A+A^2-I$$

$$A^3=\begin{bmatrix}1 & 0 & 0\\ 2 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$$

$$A^4=A^2+A^2-I=2A^2-I$$

$$A^4=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 2 & 0 & 1\end{bmatrix} \text{ and } A^5=\begin{bmatrix}1 & 0 & 0\\ 3 & 0 & 1\\ 2 & 1 & 0\end{bmatrix}$$

$$A^{50}=\begin{bmatrix}1 & 0 & 0\\ 25 & 1 & 0\\ 25 & 0 & 1\end{bmatrix}$$

Sum of elements $=53$

Question30

Let the system of equations :

$$2x+3y+5z=9$$
$$7x+3y-2z=8$$
$$12x+3y-(4+\lambda)z=16-\mu$$

have infinitely many solutions. Then the radius of the circle centred at (λ,μ) and touching the line $4x=3y$ is :

JEE Main 2025 (Online) 7th April Morning Shift

Options:

A. $\frac{7}{5}$

B. $\frac{21}{5}$

C. 7

D. $\frac{17}{5}$

Answer: A

Solution:

$$\Delta = \begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 12 & 3 & -(4+\lambda) \end{vmatrix}$$
$$= 2(-12-3\lambda+6) - 3(-28-7\lambda+24) + 5(21-36)$$
$$= -12-6\lambda+12+21\lambda-75$$
$$= 15\lambda - 75$$
$$\Rightarrow 15\lambda - 75 = 0$$
$$\Rightarrow \lambda = 5$$
$$\Delta_1 = \begin{vmatrix} 9 & 3 & 5 \\ 8 & 3 & -2 \\ 16-\mu & 3 & -9 \end{vmatrix}$$
$$= 9(-27+6) - 3(-72+32-2\mu) + 5(24-48+3\mu)$$
$$= -189+120+6\mu-120+15\mu$$
$$= 21\mu - 189 = 0$$
$$\Rightarrow \mu = 9$$

$$\therefore r = \left| \frac{4(5)-3(9)}{\sqrt{(4)^2+(3)^2}} \right|$$
$$r = \frac{7}{5}$$

Question31

Let A be a 3×3 matrix such that $|\text{adj}(\text{adj}(\text{adj A}))| = 81$.

If $S = \left\{ n \in \mathbb{Z} : (|\text{ adj}(\text{adj } A)|)^{\frac{(n-1)^2}{2}} = |A|^{(3n^2-5n-4)} \right\}$, then $\sum_{n \in S} \left| A^{(n^2+n)} \right|$ is equal to :

JEE Main 2025 (Online) 7th April Morning Shift

Options:

A. 820

B. 866

C. 750

D. 732

Answer: D

Solution:

$$|\operatorname{adj}(\operatorname{adj}(\operatorname{adj} A))| = 81$$
$$= |A|^{(n-1)^3} = (3)^4 \Rightarrow |A|^8 = 3^4 \Rightarrow |A| = 3^{1/2}$$
$$|\operatorname{adj}(\operatorname{adj} A)|^{\frac{(n-1)^2}{2}} = |A|^{(3n^2-5n-4)}$$
$$\left[|A|^{(n-1)^2}\right]^{\frac{(n-1)^2}{2}} = |A|^{3n^2-5n-4}$$
$$|A|^{2(n-1)^2} = |A|^{3n^2-5n-4}$$
$$\Rightarrow 2(n-1)^2 = 3n^2 - 5n - 4$$
$$n^2 - n - 6 = 0$$
$$\Rightarrow n = -2, 3$$
$$\sum_{x \leftarrow 5} \left|A^{n^2+n}\right| = \left|A^2\right| + \left|A^{12}\right|$$
$$= 3 + 3^6 = 732$$

Question32

Let the system of equations

x + 5y - z = 1

4x + 3y - 3z = 7

24x + y + λz = μ

λ, μ ∈ ℝ, have infinitely many solutions. Then the number of the solutions of this system,

if x, y, z are integers and satisfy 7 ≤ x + y + z ≤ 77, is :

JEE Main 2025 (Online) 7th April Evening Shift

Options:

A.

B.

5

C.

3

D.

6

Answer: C

Solution:

For infinitely many solution

$\Delta = 0$

$$\begin{vmatrix} 1 & 5 & -1 \\ 4 & 3 & -3 \\ 24 & 1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 1(3\lambda + 3) - 5(4\lambda + 72) - 1(4 - 72) = 0$

$\Rightarrow -17\lambda + 3 - 4 \times 72 - 4 = 0$

$\Rightarrow 17\lambda = -289$

$\Rightarrow \lambda = -17$

$\Delta 1 = 0$

$$\Rightarrow \begin{vmatrix} 1 & 5 & -1 \\ 7 & 3 & -3 \\ \mu & 1 & -17 \end{vmatrix} = 0$$

$\Rightarrow 1(-51 + 3) - 5(-119 + 3\mu) - 1(7 - 3\mu) = 0$

$\Rightarrow -48 + 595 - 15\mu - 7 + 3\mu = 0$

$\Rightarrow 12\mu = 540$

$\mu = 45$

$x + 5y - z = 1$

$4x + 3y - 3z = 7$

$24x + y - 17z = 45$

Let $z = 1$

$x + 5y = 1 + \lambda] \times 4$

$4x + 3y = 7 + 3\lambda$

$\frac{4x+20y=4+4\lambda}{-17y=3-\lambda}$

$$y = \frac{\lambda - 3}{17}, x = 1 + \lambda - \frac{5\lambda - 15}{17}$$
$$= \frac{32 - 12\lambda}{17}$$
$$7 \leq \frac{\lambda - 3}{17} + \frac{32 + 12\lambda}{17} + \lambda \leq 77$$
$$7 \leq \frac{30\lambda + 29}{17} \leq 77$$
$$3 \leq \lambda \leq 42$$
$$\lambda = 3, 20, 37$$

Question33

Let $A = \begin{bmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{bmatrix}$.

If $\det(\text{adj}\,(\text{adj}\,(3A))) = 2^m \cdot 3^n$, $m, n \in \mathbb{N}$, **then** $m+n$ **is equal to**

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

22

B.

20

C.

24

D.

26

Answer: C

Solution:

$$|A| = \begin{vmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{vmatrix}$$

$C_3 \to C_3 - C_2 - C_1 \times \frac{q}{2}$

Then $C_3 \to C_2 - C_1 X \left(1 + \frac{p}{2}\right)$

$$\Rightarrow |A| = \begin{vmatrix} 2 & 0 & 0 \\ 4 & 2 & 2+p \\ 6 & 6 & 8+3p \end{vmatrix}$$

$\Rightarrow |A| = 2(16 + 6p - 12 - 6p) = 8 = 2^3$

$|\operatorname{adj}(\operatorname{adj}(3A))| = |3A|^{(3-1)^2} = |3A|^4$

$= \left(3^3 |A|\right)^4 = \left(3^3 \times 2^3\right)^4 = 2^{12} \times 3^{12}$

$\Rightarrow m + n = 24$

Question34

Let *α* be a solution of $x^2 + x + 1 = 0$, and for some *a* and *b* in

R, $\begin{bmatrix} 4 & a & b \end{bmatrix} \begin{bmatrix} 1 & 16 & 13 \\ -1 & -1 & 2 \\ -2 & -14 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$**. If** $\frac{4}{\alpha^4} + \frac{m}{\alpha^a} + \frac{n}{\alpha^b} = 3$**, then *m* + *n* is equal to _____**

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

11

B.

3

C.

8

D.

7

Answer: A

Solution:

Let α be a solution of the equation $x^2 + x + 1 = 0$. This implies that α is a cube root of unity, denoted as ω, where $\alpha^2 + \alpha + 1 = 0$.

We are given matrix equations with parameters a and b as follows:

$$[4 \quad a \quad b] \begin{bmatrix} 1 & 16 & 13 \\ -1 & -1 & 2 \\ -2 & -14 & -8 \end{bmatrix} = [0 \quad 0 \quad 0]$$

Solving these equations:

$[4 - a - 2b, 64 - a - 14b, 52 + 2a - 8b] = [0, 0, 0]$
$\Rightarrow a + 2b = 4$
$\Rightarrow a + 14b = 64$

From these, solve for a and b:

$12b = 60 \Rightarrow b = 5$
$a = -6$

We now substitute a and b into the given function:

$\frac{4}{\alpha^4} + \frac{m}{\alpha^a} + \frac{n}{\alpha^b} = 3$

We substitute $\alpha = \omega$ and simplify:

$\frac{4}{\omega} + \frac{m}{1} + \frac{n}{\omega^2} = 3$
$\Rightarrow 4\omega^2 + m + n\omega = 3$

Substitute $\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $\omega^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$:

$$4\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) + m + n\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 3$$
$$\Rightarrow -2 + m - \frac{n}{2} = 3$$
and $\frac{-4\sqrt{3}}{2} + \frac{n\sqrt{3}}{2} = 0$

From the above, solve for n and m:

$n = 4$
$m = 7$

Thus, $m + n = 11$.

Question35

Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given below are two statements :

Statement I: $f(-x)$ is the inverse of the matrix $f(x)$.

Statement II: $f(x)f(y) = f(x+y)$.

In the light of the above statements, choose the correct answer from the options given below

[27-Jan-2024 Shift 1]

Options:

A.

Statement I is false but Statement II is true

B.

Both Statement I and Statement II are false

C.

Statement I is true but Statement II is false

D.

Both Statement I and Statement II are true

Answer: D

Solution:

$$f(-x)=\begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(x)\cdot f(-x)=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

Hence statement- I is correct

Now, checking statement II

$$f(y)=\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(x)\cdot f(y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\Rightarrow f(x)\cdot f(y)=f(x+y)$

Hence statement-II is also correct.

Question36

Let A be a 2 × 2 real matrix and I be the identity matrix of order 2 . If the roots of the equation |A − xI| = 0 be -1 and 3 , then the sum of the diagonal elements of the matrix A^2 is.....

[27-Jan-2024 Shift 2]

Answer: 10

Solution:

$|A - xI| = 0$

Roots are -1 and 3

Sum of roots $= \text{tr}(A) = 2$

Product of roots $= |A| = -3$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

We have $a + d = 2$

$ad - bc = -3$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

We need $a^2 + bc + bc + d^2$

$= a^2 + 2bc + d^2$

$= (a + d)^2 - 2ad + 2bc$

$= 4 - 2(ad - bc)$

$= 4 - 2(-3)$

$= 4 + 6$

$= 10$

Question37

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ and $|2A|^3 = 2^{21}$ where $\alpha, \beta \in Z$, Then a value of α is

[29-Jan-2024 Shift 1]

Options:

A.

3

B.

5

C.

17

D.

9

Answer: B

Solution:

$|A| = \alpha^2 - \beta^2$

$|2A|^3 = 2^{21} \Rightarrow A| = 2^4$

$\alpha^2 - \beta^2 = 16$

$(\alpha+\beta)(\alpha-\beta) = 16 \Rightarrow \alpha = 4 \text{ or } 5$

Question38

Let A be a square matrix such that $AA^T = I$. Then $\frac{1}{2}A[(A+A^T)^2 + (A-A^T)^2]$ is equal to

[29-Jan-2024 Shift 1]

Options:

A.

$A^2 + I$

B.

$A^3 + I$

C.

$A^2 + A^T$

D.

$A^3 + A^T$

Answer: D

Solution:

$AA^T = I = A^TA$

On solving given expression, we get

$$\frac{1}{2}A[A^2 + (A^T)^2 + 2AA^T + A^2 + (A^T)^2 - 2AA^T]$$

$$= A[A^2 + (A^T)^2] = A^3 + A^T$$

Question39

Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$. The sum of the prime factors of $|P^{-1}AP - 2I|$ is equal to

[29-Jan-2024 Shift 2]

Options:

A.

26

B.

27

C.

66

D.

23

Answer: A

Solution:

$$|P^{-1}AP-2I| = |P^{-1}AP-2P^{-1}P|$$

$$= |P^{-1}(A-2I)P|$$

$$= |P^{-1}|\,|A-2I|\,|P|$$

$$= |A-2I|$$

$$= \begin{vmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{vmatrix} = 69$$

So, Prime factor of 69 is 3 & 23

So, sum = 26

Question40

Let R = $\begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$ **be a non-zero 3 × 3 matrix, where** $x\sin\theta = y\sin\left(\theta+\frac{2\pi}{3}\right) = z\sin\left(\theta+\frac{4\pi}{3}\right) \neq 0, \theta \in (0, 2\pi).$ **For a square matrix M, let trace (M) denote the sum of all the diagonal entries of M. Then, among the statements:**

(I) Trace (R) = 0

(II) If trace(adj (adj(R)) = 0, then R has exactly one non-zero entry.

[30-Jan-2024 Shift 2]

Options:

A.

Both (I) and (II) are true

B.

Neither (I) nor (II) is true

C.

Only (II) is true

D.

Only (I) is true

Answer: C

Solution:

$$x\sin\theta = y\sin\left(\theta + \frac{2\pi}{3}\right) = z\sin\left(\theta + \frac{4\pi}{3}\right) \neq 0$$

$\Rightarrow x, y, z \neq 0$

Also,

$$\sin\theta + \sin\left(\theta + \frac{2\pi}{3}\right) + \sin\left(\theta + \frac{4\pi}{3}\right) = 0 \ \forall\theta \in R$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

$\Rightarrow xy + yz + zx = 0$

(i) Trace (R) $= x + y + z$

If $x + y + z = 0$ and $xy + yz + zx = 0$

$\Rightarrow x = y = z = 0$

Statement (i) is False

(ii) Let P : trace (Adj(Adj(R))) $= 0$

Q : R has exactly one non zero entry

if P is false then $P \longrightarrow Q$ is always true

Statement (ii) is True

Question41

Let A be a 3×3 matrix and $\det(A) = 2$. If

$n = \det(\underbrace{\text{adj}(\text{adj}(\ldots\ldots(\text{adj}\, A)))}_{2024 - \text{ times}})$

Then the remainder when n is divided by 9 is equal to_____

[31-Jan-2024 Shift 2]

Answer: 7

Solution:

$|A| = 2$

$\underbrace{\text{adj}(\text{adj}(\text{adj}\ldots.(a)))}_{2024-\text{ times}} = |A|^{(n-1)^{2024}}$

$= |A|^{2024}$

$= 2^{2^{2024}}$

$2^{2024} = (2^2)2^{2022} = 4(8)^{674} = 4(9-1)^{674}$

$\Rightarrow 2^{2024} \equiv 4(\text{mod}\,9)$

$\Rightarrow 2^{2024} \equiv 9m + 4,\ m \leftarrow \text{ even}$

$2^{9m+4} \equiv 16 \cdot (2^3)^{3m} \equiv 16(\text{mod}\,9)$

$\equiv 7$

Question42

If $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, C = ABA^T$ and $X = A^T C^2 A$, then $\det X$ is equal to :

[1-Feb-2024 Shift 1]

Options:

A.

243

B.

729

C.

27

D.

891

Answer: B

Solution:

$$A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix} \Rightarrow \det(A) = 3$$

$$B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow \det(B) = 1$$

Now $C = ABA^T \Rightarrow \det(C) = (\det(A))^2 x \det(B)$

$|C| = 9$

Now $|X| = |A^T C^2 A|$

$= |A^T| \, |C|^2 \, |A|$

$= |A|^2 \, |C|^2$

$= 9 \times 81$

$= 729$

Question43

Let $A = I_2 - 2MM^T$, where M is real matrix of order 2×1 such that the relation $M^TM = I_1$ holds. If λ is a real number such that the relation $AX = \lambda X$ holds for some non-zero real matrix X of order 2×1, then the sum of squares of all possible values of λ is equal to :

[1-Feb-2024 Shift 2]

Answer: 2

Solution:

$$A = I_2 - 2MM^T$$

$$A^2 = (I_2 - 2MM^T)(I_2 - 2MM^T)$$

$$= I_2 - 2MM^T - 2MM^T + 4MM^TMM^T$$

$$= I_2 - 4MM^T + 4MM^T$$

$$= I_2$$

$$AX = \lambda X$$

$$A^2X = \lambda AX$$

$$X = \lambda(\lambda X)$$

$$X = \lambda^2 X$$

$$X(\lambda^2 - 1) = 0$$

$$\lambda^2 = 1$$

$$\lambda = \pm 1$$

Sum of square of all possible values $= 2$

Question44

If A and B are two non-zero n × n matrics such that $A^2 + B = A^2B$, then [24-Jan-2023 Shift 1]

Options:

A. AB = I

B. $A^2B = I$

C. $A^2 = I$ or B = I

D. $A^2B = BA^2$

Answer: D

Solution:

Solution:

$A^2 + B = A^2B$

$(A^2 - I)(B - I) = I$

$A^2 + B = A^2B$

$A^2(B - I) = B$

$A^2 = B(B - I)^{-1}$

$A^2 = B(A^2 - I)$

$A^2 = BA^2 - B$

$A^2 + B = BA^2$

$A^2B = BA^2$

Question45

The number of square matrices of order 5 with entries from the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1 , is [24-Jan-2023 Shift 2]

Options:

A. 225

B. 120

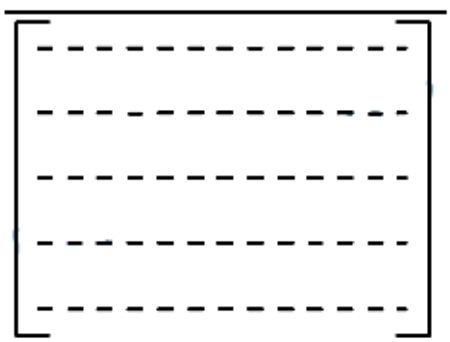
C. 150

D. 125

Answer: B

Solution:

Solution:



In each row and each column exactly one is to be placed -

∴ No. of such matrices $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Alternate :

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \rightarrow 5 \text{ ways} \\ \rightarrow 4 \text{ ways} \\ \rightarrow 3 \text{ ways} \\ \rightarrow 2 \text{ ways} \\ \rightarrow 1 \text{ ways} \end{matrix}$$

Step-1 : Select any 1 place for 1 's in row 1. Automatically some column will get filled with 0 's.
Step-2 : From next now select 1 place for 1 's. Automatically some column will get filled with 0 's. ⇒ Each time one less place will be available for putting 1's.
Repeat step-2 till last row.
Req. ways $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Question46

Let A be a 3×3 matrix such that $|\text{adj}(\text{adj}(\text{adj } A))| = 12^4$. Then $\left|A^{-1} \text{ adj } A\right|$ is equal to

[24-Jan-2023 Shift 2]

Options:

A. $2\sqrt{3}$

B. $\sqrt{6}$

C. 12

D. 1

Answer: A

Solution:

Solution:
Given $|\text{adj}(\text{adj}(\text{adj} \cdot A))| = 12^4$
$\Rightarrow |A|^{(n-1)^3} = 12^4$
We are asked
$|A^{-1} \cdot \text{adj } A|$
$= |A^{-1}| \cdot |\text{adj } A|$
$= \frac{1}{|A|} \cdot |A|^{3-1}$
$= |A| = 2\sqrt{3}$

Question47

Let x, y, z > 1 and

$$\mathbf{A} = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}.$$

Then $\left|\text{adj}(\text{adj } A^2)\right|$ is equal to

[25-Jan-2023 Shift 1]

Options:

A. 6^4

B. 2^8

C. 4^8

D. 2^4

Answer: B

Solution:

Solution:

$$|A| = \frac{1}{\log x \cdot \log y \cdot \log z} \begin{vmatrix} \log x & \log y & \log z \\ \log x & 2\log y & \log z \\ \log x & \log y & 3\log z \end{vmatrix} = 2$$

$$\Rightarrow \left|\text{adj}(\text{adj } A^2)\right| = \left|A^2\right|^4 = 2^8$$

Question48

Let A = $\begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ **and B =** $\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$**, where i =** $\sqrt{-1}$**. If M =** $A^T BA$**, then the inverse of the matrix** $AM^{2023}A^T$ **is [25-Jan-2023 Shift 2]**

Options:

A. $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Answer: D

Solution:

Solution:

$$AA^T = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}}cc\frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$$

.
.
.

$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

$M = A^T BA$

$M^2 = M \cdot M = A^T BA\, A^T BA = A^T B^2 A$

$M^3 = M^2 \cdot M = A^T B^2 AA^T BA = A^T B^3 A$

.

.

.

$M^{2023} = \ldots\ldots\ldots\ldots \cdot A^T B^{2023} A$

$AM^{2023}A^T = AA^T B^{2023} AA^T = B^{2023}$

$= \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

Inverse of $(AM^{2023}A^T)$ is $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Question49

Let A, B, C be 3 × 3 matrices such that A is symmetric and B and C are skew-symmetric.
Consider the statements
(S1)$A^{13}B^{26} - B^{26}A^{13}$ is symmetric
(S2) $A^{26}C^{13} - C^{13}A^{26}$ is symmetric
Then.
[25-Jan-2023 Shift 2]

Options:

A. Only S2 is true

B. Only S1 is true

C. Both S1 and S2 are false

D. Both S1 and S2 are true

Answer: A

Solution:

Solution:

Given, $A^T = A, B^T = -B, C^T = -C$

Let $M = A^{13}B^{26} - B^{26}A^{13}$

Then, $M^T = (A^{13}B^{26} - B^{26}A^{13})^T$

$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$

$= (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26}$
$= B^{26}A^{13} - A^{13}B^{26} = -M$
Hence, M is skew symmetric
Let, $N = A^{26}C^{13} - C^{13}A^{26}$
then, $N^T = (A^{26}C^{13})^T - (C^{13}A^{26})^T$
$= -(C)^{13}(A)^{26} + A^{26}C^{13} = N$
Hence, N is symmetric.
∴ Only S2 is true.

Question50

Let α and β be real numbers. Consider a 3 × 3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then
[29-Jan-2023 Shift 1]

Options:

A. $\alpha = 1$

B. $\alpha = 4$

C. $\beta = 8$

D. $\beta = -8$

Answer: D

Solution:

Solution:
$A^2 = 3A + \alpha I$
$A^3 = 3A^2 + \alpha A$
$A^3 = 3(3A + \alpha I) + \alpha A$
$A^3 = 9A + \alpha A + 3\alpha I$
$A^4 = (9 + \alpha)A^2 + 3\alpha A$
$= (9 + \alpha)(3A + \alpha I) + 3\alpha A$
$= A(27 + 6\alpha) + \alpha(9 + \alpha)$
$\Rightarrow 27 + 6\alpha = 21 \Rightarrow \alpha = -1$
$\Rightarrow \beta = \alpha(9 + \alpha) = -8$

Question51

The set of all values of $t \in \mathbb{R}$, for which the matrix

$$\begin{bmatrix} e^{t} & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^{t} & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^{t} & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$

is invertible, is
[29-Jan-2023 Shift 2]

Options:

A. $\left\{ (2k+1)\frac{\pi}{2}, k \in \mathbb{Z} \right\}$

B. $\left\{ k\pi + \frac{\pi}{4}, k \in \mathbb{Z} \right\}$

C. $\{k\pi, k \in \mathbb{Z}\}$

D. $\mathbb{R}$

Answer: D

Solution:

Solution:
If its invertible, then determinant value $\neq 0$
So,

$$\begin{vmatrix} e^{t} & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^{t} & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^{t} & e^{-t}\cos t & e^{-t}\sin t \end{vmatrix} \neq 0$$

$$\Rightarrow e^{t} \cdot e^{-t} \cdot e^{-t} \begin{vmatrix} 1 & \sin t - 2\cos t & -2\sin t - \cos t \\ 1 & 2\sin t + \cos t & \sin t - 2\cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

Applying, $R_1 \rightarrow R_1 - R_2$ then $R_2 \rightarrow R_2 - R_3$
We get

$$e^{-t} \begin{vmatrix} 0 & -\sin t - \cos t & -3\sin t + \cos t \\ 0 & 2\sin t & -2\cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

By expanding we have,

$$e^{-t} \times 1(2\sin t \cos t + 6\cos^{2} t + 6\sin^{2} t - 2\sin t \cos t) \neq 0$$

$$\Rightarrow e^{-t} \times 6 \neq 0$$

for $\forall t \in$

Question52

Let A be a symmetric matrix such that |A| = 2 and

$$\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$$

. If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to _____.

[29-Jan-2023 Shift 2]

Answer: 5

Solution:

Solution:

$$\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$$

Now $ac - b^2 = 2$ and $2a + b = 1$ and $2b + c = 2$
solving all these above equations we get

$$\frac{1-b}{2} \times \left(\frac{2-2b}{1} \right) - b^2 = 2$$

$$\Rightarrow (1-b)^2 - b^2 = 2$$

$$\Rightarrow 1 - 2b = 2$$

$$\Rightarrow b = -\frac{1}{2} \text{ and } a = \frac{3}{4} \text{ and } c = 3$$

Hence $\alpha = 3a + \frac{3b}{2} = \frac{9}{4} - \frac{3}{4} = \frac{3}{2}$

and $\beta = 3b + \frac{3c}{2} = -\frac{3}{2} + \frac{9}{2} = 3$

also $s = a + c = \frac{15}{4}$

$$\therefore \frac{\beta s}{\alpha^2} = \frac{3 \times 15}{4 \times \frac{9}{4}} = 5$$

Question53

Let $A = \begin{pmatrix} m & n \\ p & q \end{pmatrix}$, $d = |A| \neq 0$ $|A - d(\text{Adj } A)| = 0$

Then
[30-Jan-2023 Shift 1]

Options:

A. $(1 + d)^2 = (m + q)^2$

B. $1 + d^2 = (m + q)^2$

C. $(1 + d)^2 = m^2 + q^2$

D. $1 + d^2 = m^2 + q^2$

Answer: A

Solution:

Solution:

$$\text{Sol.} \quad A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}, \quad |A - d(\text{adj } A)| = 0$$

$$\Rightarrow |A - d(\text{adj } A)| = \left| \begin{bmatrix} m & n \\ p & q \end{bmatrix} - d \begin{bmatrix} q & -n \\ -p & m \end{bmatrix} \right|$$

$$= \begin{vmatrix} m - qd & n(1 + d) \\ p(1 + d) & q - md \end{vmatrix} = 0$$

$$\Rightarrow (m - qd)(q - md) - np(1 + d)^2 = 0$$
$$\Rightarrow mq - m^2d - q^2d + mqd^2 - np(1 + d)^2 = 0$$
$$\Rightarrow (mq - np) + d^2(mq - np) - d(m^2 + q^2 + 2np) = 0$$
$$\Rightarrow d + d^3 - d((m + q)^2 - 2d) = 0$$
$$\Rightarrow 1 + d^2 = (m + q)^2 - 2d$$
$$\Rightarrow (1 + d)^2 = (m + q)^2$$

Question54

If P is a 3×3 real matrix such that $P^T = aP + (a - 1)I$, where $a > 1$, then
[30-Jan-2023 Shift 2]

Options:

A. P is a singular matrix

B. $|\text{Adj } P| > 1$

C. $|\text{Adj } P| = \frac{1}{2}$

D. $|\text{Adj } P| = 1$

Answer: D

Solution:

Solution:

$P^T = aP + (a-1)I$

$\Rightarrow P = aP^T + (a-1)I$

$\Rightarrow P^T - P = a(P - P^T)$

$\Rightarrow P = P^T, \quad \text{as} \quad a \neq -1$

Now, $P = aP + (a-1)I$

$\Rightarrow P = -I \Rightarrow |P| = 1$

$\Rightarrow |\text{Adj } P| = 1$

Question55

Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to:
[31-Jan-2023 Shift 1]

Options:

A. 6144

B. 4094

C. 4097

D. 2050

Answer: C

Solution:

Solution:

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$$

$\Rightarrow A^3 = A^4 = \ldots\ldots = A$

$(A+I)^{11} = {}^{11}C_0 A^{11} + {}^{11}C_1 A^{10} + \ldots . {}^{11}C_{10} A + {}^{11}C_{11} I$

$= ({}^{11}C_0 + {}^{11}C_1 + \ldots . {}^{11}C_{10})A + I$

$= (2^{11} - 1)A + I = 2047A + I$

$\therefore$ Sum of diagonal elements $= 2047(1 + 4 - 3) + 3$

$= 4094 + 3 = 4097$

Question56

Let $A = [a_{ij}]$, $a_{ij} \in Z \cap [0, 4]$, $1 \le i, j \le 2$. The number of matrices A such that the sum of all entries is a prime number $p \in (2, 13)$ is _____. [31-Jan-2023 Shift 2]

Answer: 204

Solution:

Solution:

As given $a + b + c + d = 3$ or 5 or 7 or 11

if sum $= 3$

$(1 + x + x^2 + \ldots + x^4)^4 \rightarrow x^3$

$(1 - x^5)^4 (1 - x)^{-4} \rightarrow x^3$

$\therefore {}^{4+3-1}C_3 = {}^6C_3 = 20$

If sum $= 5$

$(1 - 4x^5)(1 - x)^{-4} \rightarrow x^5$

$\Rightarrow {}^{4+5-1}C_5 - 4x^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum $= 7$

$(1 - 4x^5)(1 - x)^{-4} \rightarrow x^7$

$\Rightarrow {}^{4+5-1}C_4 - {}^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum $= 11$

$(1 - 4x^5 + 6x^{10})(1 - x)^{-4} \rightarrow x^{11}$

$\Rightarrow {}^{4+11-1}C_{11} - 4 \cdot {}^{4+6-4}C_6 + 6 \cdot {}^{4+1-1}C_1$

$= {}^{14}C_{11} - 4 \cdot {}^{9}C_{6} + 6.4 = 364 - 336 + 24 = 52$

$\therefore$ Total matrices $= 20 + 52 + 80 + 52 = 204$

Question57

Let A be a n × n matrix such that |A| = 2. If the determinant of the matrix Adj(2 . Adj(2A^{-1})). is 2^{84}, then n is equal to _____.
[31-Jan-2023 Shift 2]

Answer: 5

Solution:

Solution:

$$|\text{Adj}(2\,\text{Adj}(2A^{-1}))|$$
$$= |2\,\text{Adj}(\text{Adj}(2A^{-1}))|^{n-1}$$
$$= 2^{n(n-1)}\,|\text{Adj}(2A^{-1})|^{n-1}$$
$$= 2^{n(n-1)}\,|(2A^{-1})|^{(n-1)(n-1)}$$
$$= 2^{n(n-1)}2^{n(n-1)(n-1)}\,|A^{-1}|^{(n-1)(n-1)}$$
$$= 2^{n(n-1)+n(n-1)(n-1)}\,\frac{1}{|A|^{(n-1)^2}}$$
$$= \frac{2^{n(n-1)+n(n-1)(n-1)}}{2^{(n-1)^2}}$$
$$= 2^{n(n-1)+n(n+1)^2-(n-1)^2}$$
$$= 2^{(n-1)(n^2-n+1)}$$

Now, $2^{(n-1)(n^2-n+1)}$

$2^{(n-1)(n^2-n+1)} = 2^{84}$

So, $n = 5$

Question58

If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then :
[1-Feb-2023 Shift 2]

Options:

A. $A^{30} - A^{25} = 2I$

B. $A^{30} + A^{25} + A = I$

C. $A^{30} + A^{25} - A = I$

D. $A^{30} = A^{25}$

Answer: C

Solution:

Solution:

$$A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{bmatrix}$$

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ Here $\alpha = \frac{\pi}{3}$

$$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^{30} = \begin{bmatrix} \cos 30\alpha & \sin 30\alpha \\ -\sin 30\alpha & \cos 30\alpha \end{bmatrix}$$

$$A^{30} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$A^{25} = A$

$A^{25} - A = 0$

Question59

Let $A = [a_{ij}]_{2\times 2}$, where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of | all diagonal elements of A and b = | A|. Then $3a^2 + 4b^2$ is equal to : [6-Apr-2023 shift 1]

Options:

A. 14

B. 4

C. 3

D. 7

Answer: B

Solution:

Solution:

$A^2 = I \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1 = b$

Let $A = \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix}$

$A^2 = \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} = I$

$\begin{bmatrix} \alpha^2+\beta\gamma & \alpha\beta+\beta\delta \\ \alpha\gamma+\gamma\delta & \gamma\beta+\delta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \alpha^2+\beta\gamma = 1$

$(\alpha+\delta)\beta = 0 \Rightarrow \alpha+\delta = 0 = a$

$(\alpha+\delta)\gamma = 0$

$\beta\gamma + \delta^2 = 0$

Now $3a^2 + 4b^2 = 3(0)^2 + 4(1) = 4$

Question60

Let P be a square matrix such that $P^2 = I - P$. For $\alpha, \beta, \gamma, \delta \in N$, if $P^\alpha + P^\beta = \gamma I - 29P$ and $P^\alpha - P^\beta = \delta I - 13P$, then $\alpha + \beta + \gamma - \delta$ is equal to : [6-Apr-2023 shift 2]

Options:

A. 40

B. 22

C. 24

D. 18

Answer: C

Solution:

Solution:

$P^2 = I - P$

$P^\alpha + P^\beta = \gamma I - 29P$

$P^\alpha - P^\beta = \delta I - 13P$

$P^4 = (I - P)^2 = I + P^2 - 2P$

$P^4 = I + I - P - 2P = 2I - 3P$

$P^8 = (P^4)^2 = (2I - 3P)^2 = 4I + 9P^2 - 12P$

$= 4I + 9(I - P) - 12P$

$P^8 = 13I - 21P \ldots (1)$

$P^6 = P^4 \cdot P^2 = (2I - 3P)(I - P)$

$= 2I - 5P + 3P^2$

$= 2I - 5P + 3(I - P)$

$= 5I - 8P \ldots (2)$

$(1) + (2)$

$P^8 + P^6 = 18I - 29P$

$(1) - (2)$

$P^8 - P^6 = 8I - 13P$

From (A) $\alpha = 8, \quad \beta = 6$

$\gamma = 18$

$\delta = 8$

$\alpha + \beta + \gamma - \delta = 32 - 8 = 24$

Question61

Let A = $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. If $|\text{adj}(\text{adj}(\text{adj } 2A))| = (16)^n$, then n is equal to

[8-Apr-2023 shift 1]

Options:

A. 8

B. 9

C. 12

D. 10

Answer: D

Solution:

Solution:

$|A| = 2[3] - 1[2] = 4$

$\therefore |\text{adj}(\text{adj}(\text{adj } 2A))|$

$= |2A|^{(n-1)^3} \Rightarrow |2A|^8 = 16^n$

$\Rightarrow (2^3 |A|)^8 = 16^n$

$\Rightarrow (2^3 \times 2^2)^8 = 16^n$

$= 2^{40} = 16^n$
$= 16^{10} = 16^n \Rightarrow n = 10$

Question62

Let P = $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and Q = PAP^T. If

$P^TQ^{2007}P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then 2a + b − 3c − 4d equal to

[8-Apr-2023 shift 1]

Options:

A. 2004

B. 2007

C. 2005

D. 2006

Answer: C

Solution:

Solution:

$Q = PAP^T$

$P^T \cdot Q^{2007} \cdot P = P^T \cdot Q \cdot Q \ldots Q \cdot P$

$= P^T(PAP^T)(P \cdot AP^T) \ldots (P^T)P.$

$\Rightarrow (P^TP)A(P^TP)A \ldots A(P^TP)$

$$P^T \cdot P = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} -\sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\therefore P^T \cdot Q^{2007} \cdot P = A^{2007}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$a = 1, b = 2007, c = 0, d = 1$
$2a + b - 3c - 4d = 2 + 2007 - 4 = 2005$

Question63

If $A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$, $A^{-1} = \alpha A + \beta I$ and $\alpha + \beta = -2$, then $4\alpha^2 + \beta^2 + \lambda^2$ is equal to :
[8-Apr-2023 shift 2]

Options:

A. 14

B. 12

C. 19

D. 10

Answer: A

Solution:

Solution:

$|A - xI| = 0 \Rightarrow \begin{vmatrix} 1-x & 5 \\ \lambda & 10-x \end{vmatrix} = 0 \Rightarrow x^2 - 11x + 10 - 5\lambda = 0$

$\Rightarrow (10 - 5\lambda)A^{-1} = -A + 11I$

$\therefore \alpha = \frac{-1}{10 - 5\lambda}$ and $\beta = \frac{+11}{10 - 5\lambda}$

$\alpha + \beta = -2 \Rightarrow \frac{10}{10 - 5\lambda} = -2 \Rightarrow 10 - 5\lambda = -5 \Rightarrow \lambda = 3$

$\therefore \alpha = \frac{1}{5}$ & $\beta = \frac{-11}{5}$

$\therefore 4a^2 + \beta^2 + \lambda^2 = \frac{4}{25} + \frac{121}{25} + 3^2 = 14$ Ans.

Question64

If A is a 3 × 3 matrix and |A| = 2, then $|3\ \text{adj}(|3A|A^2)|$ is equal to :
[10-Apr-2023 shift 1]

Options:

A. $3^{12} \cdot 6^{10}$

B. $3^{11} \cdot 6^{10}$

C. $3^{12} \cdot 6^{11}$

D. $3^{10} \cdot 6^{11}$

Answer: B

Solution:

Solution:

Given $|A| = 2$

Now, $|3\,\text{adj}(|3A|A^2)|$

$|3A| = 3^3 \cdot |A|$

$= 3^3 \cdot (2)$

Adj. $(|3A|A^2) = \text{adj}\{(3^3 \cdot 2)A^2\}$

$= (2.3^3)^2(\text{adj } A)^2$

$= 2^2 \cdot 3^6 \cdot (\text{adj}A)^2$

$|3\,\text{adj}(|3A|A^2)| = |2^2 \cdot 3.3^6(\text{adj } A)^2|$

$= (2^2.3^7)^3 \,|\,\text{adj}A|^2$

$= 2^6 \cdot 3^{21}(|A|^2)^2$

$= 2^6 \cdot 3^{21}(2^2)^2$

$= 2^{10} \cdot 3^{21}$

$= 2^{10} \cdot 3^{10} \cdot 3^{11}$

$|3\,\text{adj}(|3A|A^2)| = 6^{10} \cdot 3^{11}$

Question65

If $A = \frac{1}{5!6!7!}\begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$ **, then |adj(adj(2A))| is equal to**

[10-Apr-2023 shift 2]

Options:

A. 2^{16}

B. 2^8

C. 2^{12}

D. 2^{20}

Answer: A

Solution:

Solution:

$$|A| = \frac{1}{5!6!7!} 5!6!7! \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$$

$R_3 \rightarrow R_3 \rightarrow R_2$

$R_2 \rightarrow R_2 \rightarrow R_1$

$$|A| = \begin{vmatrix} 1 & 8 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} = 2$$

$| \text{adjadj}(2A) | = | 2A|^{(n-1)^2}$

$= | 2A|^4$

$= (2^3 | A|)^4$

$= 2^{12} | A |^4 \Rightarrow 2^{16}$

Question66

Let A = $\begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$, where a, c $\in \mathbb{R}$. If $A^3 = A$ and the positive value of a belongs to the interval (n − 1, n], where n $\in \mathbb{N}$, then n is equal to _____.
[11-Apr-2023 shift 1]

Answer: 2

Solution:

Solution:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$A^3 = A$

$$A^2 = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & 1 & 2+3c \end{bmatrix}$$

$$A^3 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & a & 2+3c \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 2ac+3 & a+2+3c & 2a+4+6c \\ a(a+3c)+2a & 3+2ac & 6+3a+9c \\ a+2+3c & ac+c(2+3c) & 2ac+3 \end{bmatrix}$$

Given $A^3 = A$

$2ac + 3 = 0 \ldots (1)$ and $a + 2 + 3c = 1$

$a + 1 + 3c = 0$

$a + 1 - \frac{9}{2a} = 0$

$2a^2 + 2a - 9 = 0$

$f(1) < 0, f(2) > 0$

$a \in (1, 2]$

$n = 2$

Question67

Let $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$. If $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to
[12-Apr-2023 shift 1]

Options:

A. 50

B. 75

C. 125

D. 100

Answer: D

Solution:

Solution:

$$\text{Let } C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}, D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$DC = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$B = CAD$

$B^n = (CAD)(CAD)(CAD)\ldots\ldots(CAD)_{\text{n-times}}$

$\Rightarrow B^n = CA^nD \quad \ldots(1)$

$$A^3 = \begin{bmatrix} 1 & \frac{3}{51} \\ 0 & 1 \end{bmatrix}$$

Similarly $A^n = A^3 = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

$$Bn \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{n}{51}+2 \\ -1 & -\frac{n}{51}-1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{n}{51}+1 & \frac{n}{51} \\ -\frac{n}{51} & 1-\frac{n}{51} \end{bmatrix}$$

$$\sum_{n=1}^{50} B^n = \begin{bmatrix} 25-50 & 25 \\ -25 & -25-50 \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$$

Sum of the elements $= 100$

Question68

The number of symmetric matrices of order 3 , with all the entries from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, is
[13-Apr-2023 shift 1]

Options:

A. 10^9

B. 10^6

C. 9^{10}

D. 6^{10}

Answer: B

Solution:

Solution:

$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$, a, b, c, d , e, f $\in \{0, 1, 2, \ldots .9\}$, Number of matrices $= 10^6$

Question69

Let B = $\begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$, $\alpha > 2$ be the adjoint of a matrix A and $|A| = 2$.

then $[\ \alpha \ \ -2\alpha \ \ \alpha\ ]B \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix}$ is equal to

[13-Apr-2023 shift 1]

Options:

A. 16

B. 32

C. 0

D. -16

Answer: D

Solution:

Solution:

$$\text{Given, } B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$$

$|B| = 4$

$1(8-3\alpha) - 3(4-3\alpha) + \alpha(\alpha - 2\alpha) = 4$

$-\alpha^2 + 6\alpha - 8 = 0$

$\alpha = 2, 4$

Given $\alpha > 2$

So, $\alpha = 2$ is rejected

$$\begin{bmatrix} 4 & -8 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ -8 \\ 4 \end{bmatrix} = [-16]_{1\times 1}$$

Question70

Let for $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $|A| = 2$. If $|2\,\text{adj}(2\,\text{adj}(2A))| = 32^n$, then $3n + \alpha$ is equal to
[13-Apr-2023 shift 2]

Options:

A. 10

B. 9

C. 12

D. 11

Answer: D

Solution:

Solution:

$|A| = 2$

$\text{adj}(kA) = k^{m-1}\,\text{adj}A \quad \{m = \text{ order of matrix }\}$

$\text{adj}(2A) = 2^2\,\text{adj}\,A = 4\,\text{adj}(A)$

$\text{adj}(2\,\text{adj}(2A)) = \text{adj}(8\,\text{adj}\,A)$

$= 8^2 \text{ adj adj}(A)$

$|\ 2 \text{ adj } 2 \text{ adj}(2A)\ | = |2^7 \text{ adj adj}(A)|$

$= (2^7)^3\ |A|^{2^2}$

$= 2^{21}\ |A|^4$

$= 2^{21} \cdot 2^4$

$\Rightarrow 2^{25} = (32)^n$

$\Rightarrow 2^{25} = 2^{5n}$

$\Rightarrow n = 5$

$|\ A\ | = 2$

$(6-1) - 2(2\alpha - 1) + 3(\alpha - 3) = 2$

$\Rightarrow 5 - 4\alpha + 2 + 3\alpha - 9$

$\Rightarrow \alpha = -4$

$3n + \alpha = 11$

Question71

Let the determinant of a square matrix A of order m be m − n, where m and n satisfy 4m + n = 22 and 17m+ 4n = 93. If $\det(n \text{ adj}(\text{adj}(mA))) = 3^a 5^b 6^c$, then a + b + c is equal to [15-Apr-2023 shift 1]

Options:

A. 101

B. 84

C. 109

D. 96

Answer: D

Solution:

Solution:

$|\ A\ | = m - n$

$4m + n = 22$

$17m + 4n = 93$

$m = 5, n = 2$

$|\ A\ | = 3$

$|2 \text{ adj}(\text{adj } 5A))| = 2^5\ |5A|^{16}$

$= 2^5 \cdot 5^{80}\ |A|^{16}$

$= 2^5 \cdot 5^{80} \cdot 3^{16}$

$= 3^{11} \cdot 5^{80} \cdot 6^5$

$a + b + c = 96$

Question72

Let $S = \left\{ \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix} ; a, b \in \{1, 2, 3, \ldots .100\} \right\}$ and let $T_n = \{A \in S : A^{n(n+1)} = I\}$. Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is [24-Jun-2022-Shift-2]

Answer: 100

Solution:

Solution:

$$A = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix} \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -a+ab \\ 0 & b^2 \end{bmatrix}$$

$\therefore T_n = \{A \in S; A^{n(n+1)} = I\}$

$\therefore$ b must be equal to 1

$\therefore$ In this case A^2 will become identity matrix and a can take any value from 1 to 100

$\therefore$ Total number of common element will be 100 .

Question73

Let A be a 3×3 real matrix such that

$$A \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ; A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \text{ and}$$

$$\mathbf{A}\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}$$

If $X=(x_1, x_2, x_3)^T$ and I is an identity matrix of order 3 , then the system $(A-2I)X=\begin{pmatrix}4\\1\\1\end{pmatrix}$ has :
[25-Jun-2022-Shift-1]

Options:

A. no solution

B. infinitely many solutions

C. unique solution

D. exactly two solutions

Answer: B

Solution:

Solution:

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$$A = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} a+b=1 \\ d+e=1 \\ g+h=0 \end{matrix}$$

$$A = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{matrix} a+c=-1 \\ d+f=1 \\ g+i=0 \end{matrix}$$

$$A = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \Rightarrow \begin{matrix} rc=1 \\ f=1 \\ i=2 \end{matrix}$$

Solving will get

$a=-2,\ b=3,\ c=1,\ d=-1,\ e=2, f=1,\ g=-1,\ h=1,\ i=2$

$$A = \begin{bmatrix} -2 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix} \Rightarrow A = 2I = \begin{bmatrix} -4 & 3 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$(A-2I)x = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow -4x_1 + 3x_2 + x_3 = 4 \ldots.$ (i)

$-x_1 + x_3 = 1$

$-x_1 + x_2 = 1 \ldots$ (ii)

So 3(iii) + (ii) = (i)

$\therefore$ Infinite solution

Question74

Let $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. If M and N are two matrices given by $M = \sum_{k=1}^{10} A^{2k}$ and $N = \sum_{k=1}^{10} A^{2k-1}$ then MN^2 is :
[25-Jun-2022-Shift-1]

Options:

A. a non-identity symmetric matrix

B. a skew-symmetric matrix

C. neither symmetric nor skew-symmetric matrix

D. an identity matrix

Answer: A

Solution:

Solution:

$$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I$$

$$M = A^2 + A^4 + A^6 + \ldots\ldots + A^{20}$$

$$= -4I + 16I - 64I + \ldots\ldots \text{ upto 10 terms}$$

$$= -I[4 - 16 + 64\ldots. + \text{upto 10 terms }]$$

$$= -I.4\left[\frac{(-4)^{10} - 1}{-4 - 1}\right] = \frac{4}{5}(2^{20} - 1)I$$

$$N = A^1 + A^3 + A^5 + \ldots\ldots + A^{19}$$

$$= A - 4A + 16A + \ldots\ldots \text{ upto 10 terms}$$

$$= A\left(\frac{(-4)^{10} - 1}{-4 - 1}\right) = -\left(\frac{2^{20} - 1}{5}\right)A$$

$$N^2 = \frac{(2^{20} - 1)^2}{2^5}A^2 = \frac{-4}{24}(2^{20} - 1)^2 I$$

$$MN^2 = \frac{-16}{125}(2^{20} - 1)^3 I = KI \quad (K \neq \pm 1)$$

$$(MN^2)^T = (KI)^T = KI$$

$\therefore$A is correct

Question75

Let A be a 3 × 3 matrix having entries from the set {−1, 0, 1}. The number of all such matrices A having sum of all the entries equal to 5 , is
[25-Jun-2022-Shift-1]

Answer: 414

Solution:

Solution:

Case-I:

$1 \rightarrow 7$ times

and $-1 \rightarrow 2$ times

number of possible marrix $= \frac{9!}{7!2!} = 36$

Case-II:

$1 \rightarrow 6$ times,

$-1 \rightarrow 1$ times

and $0 \rightarrow 2$ times

number of possible marrix $= \frac{9!}{6!2!} = 252$

Case-III:

$1 \rightarrow 5$ times

and $0 \rightarrow 4$ times

number of possible marrix $= \frac{9!}{5!4!} = 126$

Hence total number of all such matrix $A = 414$

Question76

Let A = $\begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}$ and B = $\begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$. Then the number of elements in the set $\{(n, m) : n, m \in \{1, 2, \ldots\ldots, 10\}$. and $.nA^n + mB^m = 1\}$ is___ [25-Jun-2022-Shift-2]

Answer: 1

Solution:

Solution:

$$A^2 = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} = A$$

$\Rightarrow A^K = A, K \in I$

$$B^2 = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = B$$

So, $B^K = B, K \in I$

$nA^n + mB^m = nA + mB$

$$= \begin{bmatrix} 2n-2n \\ n-n \end{bmatrix} + \begin{bmatrix} -m & 2m \\ -m & 2m \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, $2n - m = 1, -n + m = 0, 2m - n = 1$

So, $(m, n) = (1, 1)$

Question77

Let A be a 3 × 3 invertible matrix. If |adj(24A)| = | adj(3 adj(2A))|, then $|A|^2$ is equal to : [26-Jun-2022-Shift-1]

Options:

A. 6^6

B. 2^{12}

C. 2^6

D. 1

Answer: C

Solution:

Solution:

We know, $|\text{adj}\,A| = |A|^{n-1}$

Now, $|adj24A| = |adj3(adj2A)|$

$\Rightarrow |24A|^{3-1} = |3adj2A|^{3-1}$

$\Rightarrow |24A|^2 = |3adj2A|^2$

Also, we know, $|KA| = K^n|A|$

$\Rightarrow((24)^2)^2|A|^2 = ((3)^3)^2|adj2A|^2$

$\Rightarrow(24)^6|A|^2 = 3^6\cdot(|2A|^{3-1})^2$

$\Rightarrow(24)^6|A|^2 = 3^6\cdot|2A|^4$

$\Rightarrow(24)^6|A|^2 = 3^6\cdot(2^3)^4\cdot|A|^4$

$\Rightarrow 3^6\cdot 8^6\cdot|A|^2 = 3^6\cdot 8^4\cdot|A|^4$

$\Rightarrow 8^2 = |A|^2$

$\Rightarrow |A|^2 = 64 = 2^6$

Question78

Let $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $Y = \alpha I + \beta X + \gamma X^2$ **and**

$Z = \alpha^2 I - \alpha\beta X + (\beta^2 - \alpha\gamma)X^2$, $\alpha, \beta, \gamma \in R$.

If $Y^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$, **then** $(\alpha - \beta + \gamma)^2$ **is equal to___**

[26-Jun-2022-Shift-2]

Answer: 100

Solution:

Solution:

$$\because X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore X^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore Y = \alpha I + \beta X + \gamma X^2 \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}$$

$$\because Y \,.\, Y^{-1} = I$$

$$\therefore \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{\alpha}{5} & \frac{\beta-2\alpha}{5} & \frac{\alpha-2\beta+\gamma}{5} \\ 0 & \frac{\alpha}{5} & \frac{\beta-2\alpha}{5} \\ 0 & 0 & \frac{\alpha}{5} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \alpha = 5, \beta = 10, \gamma = 15$$

$$\therefore (\alpha - \beta + \gamma)^2 = 100$$

Question79

The positive value of the determinant of the matrix A, whose

Adj(Adj(A)) = $\begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix}$ **, is____**

[27-Jun-2022-Shift-1]

Answer: 14

Solution:

Solution:

$|\text{adj}(\text{adj}(A))| = |A|^{2^2} = |A|^4$

$\therefore |A|^4 = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix}$

$= (14)^3 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$

$= (14)^3(3 - 2(-5) - 1(-1))$

$|A|^4 = (14)^4 \Rightarrow |A| = 14$

Question80

Let A and B be two 3 × 3 matrices such that AB = I and $|A| = \frac{1}{8}$. Then |adj(B adj(2A))| is equal to

[27-Jun-2022-Shift-2]

Options:

A. 16

B. 32

C. 64

D. 128

Answer: C

Solution:

Solution:
A and B are two matrices of order 3×3.
and $AB = I$,
$|A| = \frac{1}{8}$
Now, $|A|\,|B| = 1$
$|B| = 8$
$\therefore |\text{adj}\left(B(\text{adj}(2A))\right)| = \left|B(\text{adj}(2A))\right|^{2.}$
$= |B|^2\,|\text{adj}(2A)|^2$
$= 2^6\,|2A|^{2\times 2}$
$= 2^6 \cdot 2^{12} \cdot \frac{1}{2^{12}} = 64$

Question81

Let A be a matrix of order 2 × 2, whose entries are from the set {0, 1, 2, 3, 4, 5}. If the sum of all the entries of A is a prime number p, 2 < p < 8, then the number of such matrices A is____
[27-Jun-2022-Shift-2]

Answer: 180

Solution:

Solution:

∵ Sum of all entries of matrix A must be prime p such that $2 < p < 8$ then sum of entries may be 3,5 or 7 .
If sum is 3 then possible entries are $(0, 0, 0, 3)$, $(0, 0, 1, 2)$ or $(0, 1, 1, 1)$.
∴ Total number of matrices $= 4 + 4 + 12 = 20$
If sum of 5 then possible entries are
$(0, 0, 0, 5)$, $(0, 0, 1, 4)$, $(0, 0, 2, 3)$, $(0, 1, 1, 3)$, $(0, 1, 2, 2)$ and $(1, 1, 1, 2)$.
∴ Total number of matrices $= 4 + 12 + 12 + 12 + 12 + 4 = 56$
If sum is 7 then possible entries are
$(0, 0, 2, 5)$, $(0, 0, 3, 4)$, $(0, 1, 1, 5)$, $(0, 3, 3, 1)$, $(0, 2, 2, 3)$, $(1, 1, 1, 4)$, $(1, 2, 2, 2)$, $(1, 1, 2, 3)$ and $(0, 1, 2, 4)$.
Total number of matrices with sum $7 = 104$
∴ Total number of required matrices
$= 20 + 56 + 104 = 180$

Question82

Let A be a matrix of order 3×3 and $\det(A) = 2$. Then $\det(\det(A)\,\text{adj}(5\,\text{adj}(A^3)))$ is equal to [28-Jun-2022-Shift-1]

Options:

A. 512×10^6

B. 256×10^6

C. 1024×10^6

D. 256×10^{11}

Answer: A

Solution:

Solution:
$|A| = 2$

$$|A| = \text{adj}(5\,\text{adj}\,A^3) \Big|$$

$$= |25|A\Big|\text{adj}(\text{adj}\,A^3)\Big|$$

$$= 25^3 |A|^3 \cdot \Big|\text{adj}\,A^3\Big|^2$$

$$= 25^3 \cdot 2^3 \cdot \Big|A^3\Big|^4$$

$$= 25^3 \cdot 2^3 \cdot 2^{12} = 10^6 \cdot 512$$

Question83

Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, \ldots\ldots, 100\} : A^n = A\}$ is____
[28-Jun-2022-Shift-2]

Answer: 25

Solution:

Solution:

$$\therefore A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1+i & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix} \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix} = I$$

So $A^5 = A$, $A^9 = A$ and so on.
Clearly $n = 1, 5, 9, \ldots\ldots, 97$
Number of values of $n = 25$

Question84

The probability that a randomly chosen 2×2 matrix with all the entries from the set of first 10 primes, is singular, is equal to :
[29-Jun-2022-Shift-1]

Options:

A. $\frac{133}{10^4}$

B. $\frac{18}{10^3}$

C. $\frac{19}{10^3}$

D. $\frac{271}{10^4}$

Answer: C

Solution:

Solution:
Let matrix A is singular then $|A| = 0$
Number of singular matrix = All entries are same + only two prime number are used in matrix
$= 10 + 10 \times 9 \times 2$
$= 190$
Required probability $= \frac{190}{10^4} = \frac{19}{10^3}$

Question85

Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all i, j = 1, 2, 3. Then, the matrix $A^2 + A^3 + ... + A^{10}$ is equal to : [29-Jun-2022-Shift-1]

Options:

A. $\left(\frac{3^{10} - 3}{2} \right) A$

B. $\left(\frac{3^{10} - 1}{2} \right) A$

C. $\left(\frac{3^{10} + 1}{2} \right) A$

D. $\left(\frac{3^{10} + 3}{2} \right) A$

Answer: A

Solution:

Solution:
Given, $a_{ij} = 2^{j-i}$

Now, $A = \begin{bmatrix} 2^0 & 2^1 & 2^2 \\ 2^{-1} & 2^0 & 2^1 \\ 2^{-2} & 2^{-1} & 2^0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+1 & 2+2+2 & 4+4+4 \\ \frac{1}{2}+\frac{1}{2}+\frac{1}{2} & 1+1+1 & 2+2+2 \\ \frac{1}{4}+\frac{1}{4}+\frac{1}{4} & \frac{1}{2}+\frac{1}{2}+\frac{1}{2} & 1+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & 12 \\ \frac{3}{2} & 3 & 6 \\ \frac{3}{4} & \frac{3}{2} & 3 \end{bmatrix}$$

$$= 3 \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

$= 3A$

Similarly, $A^3 = 3^2A$

$A^4 = 3^3A$

$\therefore A^2 + A^3 + \ldots\ldots + A^{10}$

$= 3A + 3^2A + 3^3A + \ldots\ldots + 3^9A$

$= A(3 + 3^2 + 3^3 + \ldots\ldots + 3^9)$

$= A\left(\frac{3(3^9-1)}{3-1}\right) = \frac{3(3^9-1)}{2}A = \left(\frac{3^{10}-3}{2}\right)A$

Question86

Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$. If

$B = I - {}^5C_1(\text{adj } A) + {}^5C_2(\text{adj } A)^2 - \ldots\ldots - {}^5C_5(\text{adj } A)^5$, then the sum of all

elements of the matrix B is
[29-Jun-2022-Shift-2]

Options:

A. -5

B. -6

C. -7

D. -8

Answer: C

Solution:

Solution:

Given $A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$ and

$B = I - 5_{C_1}(\text{adj } A) + 5_{C_2}(\text{adj } A)^2 - 5_{C_3}(\text{adj } A)^3 + 5_{C_4}(\text{adj } A)^4 - 5_{C_5}(\text{adj } A)^5$

$= (I - (\text{adj } A))^5$

$\text{Cofactor of } A = \begin{bmatrix} (-1)^{1+1} \cdot 2 & (-1)^{1+2} \cdot 0 \\ (-1)^{2+1} \cdot (-1) & (-1)^{2+2} \cdot 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

Transpose of cofactor of $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

$\therefore \text{adj } A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

Now, $I - \text{adj } A$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$

Now let,

$P = I - \text{adj } A = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$

$\therefore P^2 = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$$P^4 = P^2 \cdot P^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$$

$$P^5 = P^4 \cdot P = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$$

Now sum of elements $= -1 - 5 - 1 + 0 = -7$

Question87

Let $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, where α is a non-zero real number an $N = \sum_{k=1}^{49} M^{2k}$.

If $(I - M^2)N = -2I$, then the positive integral value of α is____
[29-Jun-2022-Shift-2]

Answer: 1

Solution:

Solution:

$$M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}; M^2 = \begin{bmatrix} -\alpha^2 & 0 \\ 0 & -\alpha^2 \end{bmatrix} = -\alpha^2 I$$

$$N = M^2 + M^4 + \ldots\ldots + M^{98} = [-\alpha^2 + \alpha^4 - \alpha^6 + \ldots]I$$

$$= -\alpha^2 \frac{(1 - (-\alpha^2)^{49})}{1 + \alpha^2} \cdot I$$

$$I - M^2 = (1 + \alpha^2)I$$

$$(I - M^2)N = -\alpha^2(\alpha^{98} + 1) = -2$$

$$\alpha = 1$$

Question88

Let $S = \{\sqrt{n} : 1 \leq n \leq 50 \text{ and } n \text{ is odd}\}$.

Let $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$.

If $\sum_{a \in S} \det(\text{adj } A) = 100\lambda$, then λ is equal to : ion:

[24-Jun-2022-Shift-1]

Options:

A. 218

B. 221

C. 663

D. 1717

Answer: B

Solution:

Solution:

Given, $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}_{3\times 3}$ $S = \{\sqrt{n} : 1 \le n \le 50 \text{ and } n \text{ is odd}\}$

$\therefore S = \{1, \sqrt{3}, \sqrt{5}, \sqrt{7}, ..., \sqrt{49}\}$

We know,

$|\text{adj}\, A| = |A|^{n-1}$

Here, n = order of matrix.

Here, $n = 3$

$\therefore |adjA| = |A|^{3-1} = |A|^2$

Now, $|A| = \begin{vmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{vmatrix}$

$= 1(1-0) - 0 + a(0-(-a))$

$= a^2 + 1$

$\therefore |adjA| = |A|^2 = (a^2+1)^2$

Now, $\sum_{a\in S} \det(adjA)$

$= \sum_{a\in S} (a^2+1)^2$

$= (1^2+1)^2 + ((\sqrt{3})^2+1)^2 + ((\sqrt{5})^2+1)^2 + ... + ((\sqrt{49})^2+1)^2$

$= (1^2+1)^2 + (3+1)^2 + (5+1)^2 + ... + (49+1)^2$

$= 2^2 + 4^2 + 6^2 + ... + 50^2$

$= 2^2(1^2 + 2^2 + 3^2 + ... + 25^2)$

$= 4.\frac{25.26.51}{6} = 100.221$

$\therefore 100K = 100.221$

$\Rightarrow K = 221$

Question89

Let A = $\begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$ and B = A − I . If ω = $\frac{\sqrt{3}i-1}{2}$, then the number of elements in the set {n ∈ {1, 2, ..., 100} : $A^n + (\omega B)^n = A + B$} is equal to [25-Jul-2022-Shift-1]

Answer: 17

Solution:

Solution:

Here $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$

We get $A^2 = A$ and similarly for

$B = A - I = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$

We get $B^2 = -B \Rightarrow B^3 = B$

$\therefore A^n + (\omega B)^n = A + (\omega B)^n \quad \text{for } n \in N$

For ω^n to be unity n shall be multiple of 3 and for B^n to be B. n shell be 3, 5, 7, .. . 99

$\therefore n = \{3, 9, 15, \ldots . .99\}$

Number of elements = 17

Question90

Let $A = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$ **, a, b** $\in$ **C. If for some n** $\in \mathbb{N}$, $A^n = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$

then n + a + b is equal to
[25-Jul-2022-Shift-2]

Answer: 24

Solution:

Solution:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = I + B$$

$$B^2 = \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & ab \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$B^3 = 0$

$\therefore A^n = (1+B)^n = {}^nC_0 I + {}^nC_1 B + {}^nC_2 B^2 + {}^nC_3 B^3 + \ldots$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & na & na \\ 0 & 0 & nb \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{n(n-1)ab}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & na & na + \frac{n(n-1)}{2}ab \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 48 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing we get $na = 48$, $nb = 96$ and

$na + \frac{n(n-1)}{2}ab = 2160$

$\Rightarrow a = 4,\ n = 12$ and $b = 8$

$n + a + b = 24$

Question91

Let A be a 2 × 2 matrix with det(A) = −1 and det((A + I)(Adj(A) + I)) = 4. Then the sum of the diagonal elements of A can be : [26-Jul-2022-Shift-1]

Options:

A. -1

B. 2

C. 1

D. $-\sqrt{2}$

Answer: B

Solution:

Solution:

$$|(A+I)(adjA+I)|=4$$

$$\Rightarrow |A\,\text{adj}\,A+A+\text{adj}\,A+I\;|\;=4$$

$$\Rightarrow |(A)I+A+\text{adj}\,A+I\;|\;=4$$

$$|A|=-1\Rightarrow |A+\text{adj}\,A\;|\;=4$$

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} adjA=\begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} (a+d) & 0 \\ 0 & (a+d) \end{vmatrix}=4$$

$$\Rightarrow a+d=\pm 2$$

Question92

Let A = $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ **and B =** $\begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$ **, then the value of $A'BA$ is:**
[26-Jul-2022-Shift-2]

Options:

A. 1224

B. 1042

C. 540

D. 539

Answer: D

Solution:

Solution:

$$A'BA = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix} A$$

$$= \begin{bmatrix} 9^2+12^2-15^2 & -10^2+13^2+16^2 & 11^2-14^2+17^2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= [9^2+12^2-15^2-10^2+13^2+16^2+11^2-14^2+17^2]$$
$$= [(9^2-10^2)+(11^2+12^2)+(13^2-14^2)+(16^2-15^2)+17^2]$$
$$= [-19+265+(-27)+31+289]$$
$$= [585-46] = [539]$$

Question93

The number of matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where

$a, b, c, d \in \{-1, 0, 1, 2, 3, \ldots, ., 10\}$, such that $A = A^{-1}$, is _____. [26-Jul-2022-Shift-2]

Answer: 50

Solution:

Solution:

$\because A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^2 = \begin{bmatrix} a^2+bc & b(a+d) \\ c(a+d) & bc+d^2 \end{bmatrix}$

For A^{-1} must exist $ad - bc \neq 0$..... (i)

and $A = A^{-1} \Rightarrow A^2 = I$

$\therefore a^2+bc = d^2+bc = 1$

and $b(a+d) = c(a+d) = 0$

Case I : When $a = d = 0$, then possible values of (b, c) are $(1, 1), (-1, 1)$ and $(1, -1)$ and $(-1, 1)$.

Total four matrices are possible.

Case II: When $a = -d$ then (a, d) be $(1, -1)$ or $(-1, 1)$.

Then total possible values of (b, c) are $(12+11) \times 2 = 46$.

$\therefore$ Total possible matrices $= 46+4 = 50$.

Question94

Let $A = \begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$. Let $\alpha, \beta \in \mathbb{R}$ be such that $\alpha A^2 + \beta A = 2I$. Then $\alpha + \beta$ is equal to
[27-Jul-2022-Shift-1]

Options:

A. -10

B. -6

C. 6

D. 10

Answer: D

Solution:

Solution:

$$A^2 = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -3 & -8 \\ 8 & 21 \end{bmatrix}$$

$$\alpha A^2 + \beta A = \begin{bmatrix} -3\alpha & -8\alpha \\ 8\alpha & 21\alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -2\beta & -5\beta \end{bmatrix}$$

$$= \begin{bmatrix} -3\alpha+\beta & -8\alpha+2\beta \\ 8\alpha-2\beta & 21\alpha-5\beta \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

On Comparing
$8\alpha = 2\beta, -3\alpha + \beta = 2, 21\alpha - 5\beta = 2$
$\Rightarrow \alpha = 2, \beta = 8$
So, $\alpha + \beta = 10$

Question95

Let S be the set containing all 3×3 matrices with entries from $\{-1, 0, 1\}$. The total number of matrices $A \in S$ such that the sum of all the diagonal elements of A^TA is 6 is _____.
[27-Jul-2022-Shift-1]

Answer: 5376

Solution:

Solution:
Sum of all diagonal elements is equal to sum of square of each element of the matrix.

i.e., $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ then $t_r(A \cdot A^T)$

$= a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 + c_1^2 + c_2^2 + c_3^2$

$\because a_i, b_i, c_i \in \{-1, 0, 1\}$ for $i = 1, 2, 3$

$\therefore$ Exactly three of them are zero and rest are 1 or -1.

Total number of possible matrices ${}^9C_3 \times 2^6$

$= \frac{9 \times 8 \times 7}{6} \times 64$

$= 5376$

Question96

Let $A = \begin{pmatrix} 4 & -2 \\ \alpha & \beta \end{pmatrix}$.

If $A^2 + \gamma A + 18I = O$, then det(A) is equal to _____. [27-Jul-2022-Shift-2]

Options:

A. -18

B. 18

C. -50

D. 50

Answer: B

Solution:

Solution:
Characteristic equation of A is given by

$|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & -2 \\ \alpha & \beta-\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^2 - (4 + \beta)\lambda + (4\beta + 2\alpha) = 0$

So, $A^2-(4+\beta)A+(4\beta+2\alpha)I=0$

$|A|=4\beta+2\alpha=18$

Question97

Consider a matrix A = $\begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{bmatrix}$, where α, β, γ are three distinct natural numbers. If $\frac{\det(adj(adj(adj(adj\,A))))}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}}=2^{32}\times 3^{16}$, then the number of such 3 - tuples (α, β, γ) is _____.
[27-Jul-2022-Shift-2]

Answer: 42

Solution:

Solution:

$$\det(A)=\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$$

$R_3\to R_3+R_1$

$$\Rightarrow(\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$\therefore \det(A)=(\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

Also, $\det(adj(adj(adj(adj(A)))))$

$=(\det(A))^{2^4}=(\det(A)^{16}.$

$\therefore \frac{(\alpha+\beta+\gamma)^{16}(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}}=(4.13)^{16}$

$\Rightarrow\alpha+\beta+\gamma=12$

$\Rightarrow(\alpha,\beta,\gamma)$ distinct natural triplets

$={}^{11}C_2-1-{}^3C_2(4)=55-1-12=42$

Question98

Let the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and the matrix $B_0 = A^{49} + 2A^{98}$. If $B_n = Adj(B_{n-1})$ for all $n > 1$, then $\det(B_4)$ is equal to : [28-Jul-2022-Shift-1]

Options:

A. 3^{28}

B. 3^{30}

C. 3^{32}

D. 3^{36}

Answer: C

Solution:

Solution:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Now $B_0 = A^{49} + 2A^{98} = (A^3)^{16} \cdot A + 2(A^3)^{32} \cdot A^2$

$$B_0 = A + 2A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

$|B_0| = 9$

Since, $B_n = Adj\,|B_{n-1}| \Rightarrow |B_n| = |B_{n-1}|^2$

Hence $|B_4| = |B_3|^2 = |B_2|^4 = |B_1|^8 = |B_0|^{16} = |3^2|^{16} = 3^{32}$

Question99

Let $A = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$ and $B = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}$, $\alpha, \beta \in R$. Let α_1 be the value of α which satisfies $(A+B)^2 - A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ and α_2 be the value of α which satisfies $(A+B)^2 - B^2$. Then $|\alpha_1 - \alpha_2|$ is equal to _____.
[28-Jul-2022-Shift-1]

Answer: 2

Solution:

Solution:

$$(A+B)^2 = A^2 + B^2 + AB + BA$$

$$= A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\therefore B^2 + AB + BA = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \ldots\ldots (1)$$

$$AB = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}\begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \beta-1 & 1 \\ \alpha+2\beta & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} \beta+2 & \alpha-\beta \\ 1 & -1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \beta^2+1 & \beta \\ \beta & 1 \end{bmatrix}$$

By (1) we get

$$\begin{bmatrix} \beta^2+2\beta & \alpha+1 \\ \alpha+3\beta+1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\therefore \alpha = 1\beta = 0 \Rightarrow \alpha_1 = 1$$

Similarly if $A^2 + AB + BA = 0$ then

$$\left(A^2 = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} -1 & -1-\alpha \\ 2+2\alpha & \alpha^2-2 \end{bmatrix} \right)$$

$$\begin{bmatrix} 2\beta & \alpha-\beta+1-1-\alpha \\ \alpha+2\beta+1+2+2\alpha & \alpha^2-2+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \beta = 0 \text{ and } \alpha = -1 \Rightarrow \alpha_2 = -1$$

$$\therefore |\alpha_1 - \alpha_2| = |2| = 2$$

Question100

Let A and B be any two 3 × 3 symmetric and skew symmetric matrices respectively. Then which of the following is NOT true?
[28-Jul-2022-Shift-2]

Options:

A. $A^4 - B^4$ is a smmetric matrix

B. $AB - BA$ is a symmetric matrix

C. $B^5 - A^5$ is a skew-symmetric matrix

D. $AB + BA$ is a skew-symmetric matrix

Answer: C

Solution:

Solution:
(A) $M = A^4 - B^4$
$M^T = (A^4 - B^4)^T = (A^T)^4 - (B^T)^4$
$= A^4 - (-B)^4 = A^4 - B^4 = M$
(B) $M = AB - BA$
$M^T = (AB - BA)^T = (AB)^T - (BA)^T$
$= B^T A^T - A^T B^T$
$= -BA - A(-B)$
$= AB - BA = M$
(C) $M = B^5 - A^5$
$M^T = (B^T)^5 - (A^T)^5 = -(B^5 + A^5) \neq -M$
(D) $M = AB + BA$
$M^T = (AB)^T + (BA)^T$
$= B^T A^T + A^T B^T = -BA - AB = -M$

Question101

Let A and B be two 3 × 3 non-zero real matrices such that AB is a zero matrix. Then
[29-Jul-2022-Shift-1]

Options:

A. the system of linear equations $AX = 0$ has a unique solution

B. the system of linear equations $AX = 0$ has infinitely many solutions

C. B is an invertible matrix

D. adj(A) is an invertible matrix

Answer: B

Solution:

Solution:
AB is zero matrix
$\Rightarrow |A| = |B| = 0$
So neither A nor B is invertible
If $|A| = 0$
$\Rightarrow |\text{adj } A| = 0$ so adj A
$AX = 0$ is homogeneous system and $|A| = 0$
So, it is having infinitely many solutions

Question102

The number of matrices of order 3 × 3, whose entries are either 0 or 1 and the sum of all the entries is a prime number, is _____. [29-Jul-2022-Shift-1]

Answer: 282

Solution:

Solution:
In a 3×3 order matrix there are 9 entries.
These nine entries are zero or one.
The sum of positive prime entries are $2, 3, 5$ or 7 .
Total possible matrices $= \frac{9!}{2!.7!} + \frac{9!}{3!.6!} + \frac{9!}{5!.4!} + \frac{9!}{7!.2!} = 36 + 84 + 126 + 36 = 282$

Question103

Which of the following matrices can NOT be obtained from the matrix $\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ by a single elementary row operation?

[29-Jul-2022-Shift-2]

Options:

A. $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

C. $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$

D. $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$

Answer: C

Solution:

Solution:

$A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

(1) $R_1 \to R_1 + R_2$; $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ possible

(2) $R_1 \leftrightarrow R_2$; $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ possible

(3) Option is not possible

(4) $R_2 \to R_2 + 2R_1$; $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ possible

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Question104

Let $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A - \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$. For $k \in N$, if $X'A^kX - 33$, then k is equal to _____.
[29-Jul-2022-Shift-2]

Answer: 10

Solution:

Solution:

$$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$$

$$X^T A^K X = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}^k \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

As $A^2 = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^4 = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

for $K \rightarrow \text{Even}\ A^K = \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$X^T A^K X = 33$ (This is not correct)

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 3K+1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [3K+3]$$

$\therefore 3K + 3 = 33 \therefore K = 10$

But it should be dropped as 33 is not matrix

If K is odd

$X^T A^K X = 33$

$X^T A A^{K-1} X = 33$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3k-3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\begin{bmatrix} -1 & 3 & 8 \end{bmatrix} \begin{bmatrix} 3k-2 \\ 1 \\ 1 \end{bmatrix} = [33]$$

$[-3k + 13] = [33]$

$k = 20/3$ (not possible)

Question105

Let M be any 3 × 3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven, is
24 Feb 2021 Shift 1

Answer: 540

Solution:

Solution:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2+i^2=7$

Case-I : Seven (1's) and two (0's)

$\frac{9!}{7!2!}=36$

Case-II: One (2) and three (1's) and five (0's)

$\frac{9!}{5!3!}=504$

$\therefore$ Total $=540$

Question106

If $A=\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ **and** $(I_2+A)(I_2-A)^{-1}=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$**, then**

$13(a^2+b^2)$ is ..... .

[25 Feb 2021 Shift 1]

Answer: 13

Solution:

Solution:

$A=\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2+A)(I_2-A)^{-1}=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow |(I_2+A)(I_2-A)^{-1}|=a^2+b^2$

$\Rightarrow a^2+b^2=\frac{|I_2+A|}{|I_2-A|}$ . . . (i)

Now, $I_2+A=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$$

Similarly, $I_2 - A = \begin{bmatrix} 1 & \tan\left(\frac{\theta}{2}\right) \\ -\tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$

Here, $|I_2 + A| = |I_2 - A| = \left(1 + \tan^2\left(\frac{\theta}{2}\right)\right)$

$\Rightarrow \frac{|I_2 + A|}{|I_2 - A|} = 1$ . . . (ii)

From Eqs. (i) and (ii),

$a^2 + b^2 = 1$

Now, $13(a^2 + b^2) = 13 \times 1 = 13$

Question107

If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$,

$AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is

[2021, 25 Feb. Shift-II]

Options:

A. 4

B. 1

C. 2

D. 3

Answer: B

Solution:

Solution:

Given, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix}$

Given, $AA^T = I_2$ i.e.

$$\begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1+\alpha^2 & \alpha-\alpha\beta \\ \alpha-\alpha\beta & \alpha^2+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating these matrices,

$c1 = \alpha^2 + 1$ gives, $\alpha = 0$

$\alpha(1-\beta) = 0$

$\alpha^2 + \beta^2 = 1$

Put $\alpha = 0$ in $\alpha^2 + \beta^2 = 1$, we get $0 + \beta^2 = 1$, gives $\beta = \pm 1$

where we take $\beta^4 = 1$

$\therefore \ \alpha^4 + \beta^4 = 0^4 + 1 = 1$

Question108

Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1 , then the possible number of such matrices is
[2021, 26 Feb. Shift-I]

Options:

A. 4

B. 1

C. 6

D. 12

Answer: A

Solution:

Solution:

Let A be the matrix as follows, $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$, since A is symmetric matrix.

Now, $A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$$= \begin{bmatrix} a^2+b^2 & ab+bc \\ ab+bc & b^2+c^2 \end{bmatrix}$$

Given that, diagonal entries of A^2 is 1 . i.e. $a^2+b^2+b^2+c^2=1$
or $a^2+2b^2+c^2=1$
Case $1a=0$
Then, $2b^2+c^2=1$
(a) $c=0$ gives, $b^2=\frac{1}{2}$ or $b=\pm\frac{1}{\sqrt{2}}$ $\therefore a=0, b=1/\sqrt{2}, c=0$ (2 matrices) $a=0, b=-1/\sqrt{2}, c=0$
(b) $b=0$, gives $c^2=1$ or $c=\pm1$ $\therefore a=0, b=0, c=1$
and $a=0, b=0, c=-1$ (2 matrices)
Case $2b=0$, then $a^2+c^2=1$
(a) $a=0$, then $c=\pm1$ $a=0, b=0, c=1$ and $a=0, b=0, c=-1$
This is repeated case.
(b) $c=0$, then $a=\pm1$ $a=1, b=0, c=0$ and $a=-1, b=0, c=0$ Again 2 matrices.

Thus, only acceptable matrices are as follows

$$A=\begin{bmatrix}0&0\\0&1\end{bmatrix},\begin{bmatrix}0&0\\0&-1\end{bmatrix},\begin{bmatrix}1&0\\0&0\end{bmatrix},\begin{bmatrix}-1&0\\0&0\end{bmatrix}$$

Then possible number of such matrices are 4.

Question109

If the matrix $A=\begin{bmatrix}1&0&0\\0&2&0\\3&0&-1\end{bmatrix}$ satisfies the equation

$A^{20}+\alpha A^{19}+\beta A=\begin{bmatrix}1&0&0\\0&4&0\\0&0&1\end{bmatrix}$ for some real numbers α and β, then

$\beta-\alpha$ is equal to
[2021, 26 Feb. Shift-II]

Answer: 4

Solution:

Solution:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And so on,

$$A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c So, " $A^{20} + \alpha A^{19} + \beta A$

$$= \begin{bmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20}+\alpha 2^{19}+2\beta & 0 \\ 3\alpha+2\beta & 0 & 1-\alpha-\beta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing, we get

$1-\alpha-\beta = 1$

$\Rightarrow \alpha+\beta = 0$

and $2^{20} + \alpha 2^{19} + 2\beta = 4$

$\Rightarrow 2^{20} + \alpha(2^{19}-2) = 4$ [use, $\alpha+\beta = 0$]

$\Rightarrow \alpha = \dfrac{4-2^{20}}{2^{19}-2} = -2$

$\Rightarrow \beta = 2$

$\therefore \beta - \alpha = 2-(-2) = 2+2 = 4$

Question110

Let $A+2B=\begin{bmatrix}1&2&0\\6&-3&3\\-5&3&1\end{bmatrix}$ and $2A-B=\begin{bmatrix}2&-1&5\\2&-1&6\\0&1&2\end{bmatrix}$.

If T r(A) denotes the sum of all diagonal elements of the matrix A, then T r(A) − T r(B) has value equal to
[2021, 18 March Shift-1]

Options:

A. 1

B. 2

C. 0

D. 3

Answer: B

Solution:

Solution:

Given, $A+2B=\begin{bmatrix}1&2&0\\6&-3&3\\-5&3&1\end{bmatrix}$ ... (i)

and $2A-B=\begin{bmatrix}2&-1&5\\2&-1&6\\0&1&2\end{bmatrix}$ ... (ii)

Multiply with 2 in Eq. (ii), we get

$4A-2B=\begin{bmatrix}4&-2&10\\4&-2&12\\0&2&4\end{bmatrix}$ ..... (iii)

Adding Eqs. (i) and (iii),

$5A=\begin{bmatrix}5&0&10\\10&-5&15\\-5&5&5\end{bmatrix}$

$$\therefore \ A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$\therefore \text{Tr}(A) = 1 - 1 + 1 = 1$

From Eq. (i),

$$B = \frac{1}{2}\left\{ \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{bmatrix}$$

$\therefore \text{Tr}(B) = 0 - 1 + 0 = -1$

Hence, $\text{Tr}(A) - \text{Tr}(B) = 1 - (-1) = 2$

Question111

Let I be an identity matrix of order 2×2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in N$ for which $P^n = 51 - 8P$ is equal to
[2021, 18 March Shift-II]

Answer: 6

Solution:

Solution:
Method (1)

Given, $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

Now, $P^n = 51 - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix}$

$$\Rightarrow \ P^n = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} \quad \cdots\cdots\cdot (i)$$

Now, $P^2 = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$

Again $P^3 = P \cdot P^2 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix}$ Now,

$P^6 = P^3 \cdot P^3 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix}$

$\Rightarrow \; P^6 = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} = P^n \quad [\text{from Eq. (i)} \quad]$

$\therefore \; n = 6$

Method (2)

Given $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

Characteristics equation is $|P - \lambda| = 0$

$$\begin{vmatrix} 2-\lambda & -1 \\ 5 & -3-\lambda \end{vmatrix} = 0$$

$\Rightarrow \; (2-\lambda)(-3-\lambda) + 5 = 0$

$\Rightarrow \; \lambda^2 + \lambda - 1 = 0$

By Cayley-Hamilton Theorem,

$P^2 + P - 1 = 0$

$\Rightarrow \; P^2 = 1 - P$

$\Rightarrow \; P^3 + P^2 - P = 0$

$\Rightarrow \; P^3 = P - P^2 = P - (1 - P)$ [from Eq. (i)]

$\Rightarrow \; P^3 = 2P - 1$

$\Rightarrow \; P^3 = 2P - 1$

Now, $P \cdot P^3 = P(2P - 1)$

$\Rightarrow P^4 = 2P^2 - P = 2(1 - P) - P$[from Eq. (i)]

$\Rightarrow \; P^4 = -3P + 21$

Again, $P \cdot (P^4) = P(-3P + 21)$

$\Rightarrow \; P^5 = -3P^2 + 2P$

$= -3(1 - P) + 2P$[from Eq. (i)]

$= 5P - 31$

and $P(P^5) = P(5P - 31)$

11 $\Rightarrow P^6 = 5P^2 - 3P = 5(1 - P) - 3P$

$\Rightarrow p^6 = 5l - 8P = P^n$ (given)

$\therefore \; n = 6$

Question112

Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$, $i = \sqrt{-1}$. Then, the system of linear equations

$A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has

[2021, 16 March Shift-I]

Options:

A. a unique solution.

B. infinitely many solutions.

C. no solution.

D. exactly two solutions.

Answer: C

Solution:

Solution:

Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} i^2 + i^2 & -i^2 - i^2 \\ -i^2 - i^2 & i^2 + i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2$$

$$= \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & -4-4 \\ -4-4 & 4+4 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

Similarly, $A^8 = A^4 \cdot A^4$

$$= \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 64+64 & -64-64 \\ -64-64 & 64+64 \end{bmatrix}$$

$= \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}$ Now, $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$$\begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$128x - 128y = 8$

$\Rightarrow x - y = \frac{1}{16} \quad \cdots\cdots (i)$

and $-128x + 128y = 64$

$\Rightarrow x - y = -\frac{1}{2} \quad \cdots\cdots (ii)$

From Eqs. (i) and (ii), we get these two lines are parallel. So, there will be no solution.

Question113

The total number of 3 × 3 matrices A having entries from the set (0, 1, 2, 3), such that the sum of all the diagonal entries of AA^T is 9 , is equal to
[2021, 16 March Shift-I]

Answer: 766

Solution:

Solution:
Set S : {0, 1, 2, 3}

$$\text{Let } A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} a \cdot a & a \cdot b & a \cdot c \\ b \cdot a & b \cdot b & b \cdot c \\ c \cdot a & c \cdot b & c \cdot c \end{bmatrix}$$

where $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$b = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$c = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Now, as per question

$ca \cdot a + b \cdot b + c \cdot c = 9$

$\Rightarrow (a_1^2 + a_2^2 + a_3^2) + (b_1^2 + b_2^2 + b_3^2)$

$+(c_1^2 + c_2^2 + c_3^2) = 9$

$[a_i, b_i, c_i \in S]$

$9 = (1+1+1+1+1+1+1+1+1)$

or $9 = (1+4+4+0+0+0+0+0+0)$

or $9 = (9+0+0+0+0+0+0+0+0)$

or $9 = (4+1+1+1+1+1+0+0+0)$

Total permutations in case $1 = 1$

Total permutations in case $2 = \frac{9!}{6!2!} = 252$

In case $3 = \frac{9!}{8!} = 9$

In case $4 = \frac{9!}{5!3!} = 504$

Total permutations $= 1 + 252 + 9 + 504 = 766$

Question114

Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $a = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2×1 matrices with real entries such that $A = XB$, where $X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$ and $k \in R$. If $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ and $(k^2+1)b_2^2 \neq -2b_1b_2$, then the value of k is [2021, 16 March Shift-II]

Answer: 1

Solution:

Solution:

$A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$A = XB$

$$X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$$

$$XB = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$XB = \frac{1}{\sqrt{3}}\begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$$

As, $A = XB$

So, $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$

$\sqrt{3}a_1 = b_1 - b_2 \quad \cdots\cdots\cdot (i)$

$\sqrt{3}a_2 = b_1 + kb_2 \quad \cdots\cdots\cdot (ii)$

And as given, $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$

Adding, Eq. (i) ^{2}+ Eq. (ii) ^{2},

$3a_1^2 + 3a_2^2 = (b_1 - b_2)^2 + (b_1 + kb_2)^2$

$\Rightarrow 2(b_1^2 + b_2^2)$

$= 2b_1^2 + b_2^2(k^2 + 1) + 2b_1b_2(k - 1)$

$\Rightarrow b_2^2(k^2 + 1 - 2) + 2b_1b_2(k - 1) = 0$

$\Rightarrow (k - 1)[b_2^2(k + 1) + 2b_1b_2] = 0$

So, $k = 1$

Question115

Let P = $\begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ **and A =** $\begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix}$

where, $\omega = \frac{-1 + i\sqrt{3}}{2}$**, and** I_3 **be the identity matrix of order 3 . If the determinant of the matrix** $(P^{-1}AP - I_3)^2$ **is** $\alpha\omega^2$**, then the value of** α **is equal to.....**

[16 Mar 2021 Shift 1]

Answer: 36

Solution:

Solution:

Given, $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$

$|(P^{-1}AP - I_3)|^2 = \alpha\omega^2$

$\Rightarrow \quad |(P^{-1}AP - I_3)(P^{-1}AP - I_3)| = \alpha\omega^2$

$\Rightarrow P^{-1}APP^{-1}AP - P^{-1}API_3 - I_3P^{-1}AP + I_3 \cdot I_3| = \alpha\omega^2$

$\Rightarrow \quad P^{-1}A^2P - P^{-1}AP - P^{-1}AP + I_3| = \alpha\omega^2$

$[\because PP^{-1} = I \quad \text{and} \quad IA = A]$

$\Rightarrow \quad |P^{-1}A^2P - 2P^{-1}AP + PP^{-1}| = \alpha\omega^2$

$\Rightarrow \quad |P^{-1}(A^2 - 2A + I_3)P| = \alpha\omega\omega^2$

$\Rightarrow \quad |P^{-1}||A - I_3|^2|P| = \alpha\omega^2$

$\Rightarrow \quad |P^{-1}P||A - I_3|^2 = \alpha\omega^2$

$\Rightarrow |A - I_3|^2 = \alpha\omega^2$

Consider,

$$A - I_3 = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A - I_3| = \begin{vmatrix} 1 & 7 & \omega^2 \\ -1 & -\omega - 1 & 1 \\ 0 & -\omega & -\omega \end{vmatrix}$$

On applying $C_2 \to C_2 - C_3$, we get

$$|A - I_3| = \begin{vmatrix} 1 & 7 - \omega^2 & \omega^2 \\ -1 & -\omega - 2 & 1 \\ 0 & 0 & -\omega \end{vmatrix}$$

On applying $C_2 \to C_2 - C_3$, we get

$$|A - I_3| = \begin{vmatrix} 1 & 7 - \omega^2 & \omega^2 \\ -1 & -\omega - 2 & 1 \\ 0 & 0 & -\omega \end{vmatrix}$$

$= -\omega[(-\omega - 2) - (-7 + \omega^2)]$

$= -\omega(-\omega - 2 + 7 - \omega^2)$

$= -\omega(1 - 2 + 7)$

$= -6\omega$

$|A - I_3| = -6\omega$

$|A - I_3|^2 = 36\omega^2 = \alpha\omega^2$

$\therefore \quad \alpha = 36$

Question116

If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4) + \det[A^{10} - \text{Adj}(2A)^{10}]$ is equal to .....
[17 Mar 2021 Shift 1]

Answer: 16

Solution:

Solution:

$$\text{If } A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$$

$$\det(A^4) + \det[A^{10} - [\text{Adj}(2A)]^{10}]$$

$$A \cdot A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}, A^2 = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 9 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^3 \cdot A$$

$$= \begin{bmatrix} 8 & 9 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 16 & 15 \\ 0 & 1 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 2^n & 2^n - (-1)^n \\ 0 & (-1)^n \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} 2^{10} & 2^{10} - 1 \\ 0 & 1 \end{bmatrix} \Rightarrow 2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$$

$$\text{adj}(2A) = \begin{bmatrix} -2 & -6 \\ 0 & 4 \end{bmatrix}$$

$$\text{adj}(2A) = -2\begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \left[\because x = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad \text{adj}(x) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \right]$$

$$[\text{adj}(2A)]^2 = 4\begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = 4\begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix}$$

$$[\text{adj}(2A)]^3 = 4\begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix} \times (-2)\begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = -8\begin{pmatrix} 1 & 9 \\ 0 & -8 \end{pmatrix}$$

$$[\text{adj}(2A)]^3 = 4\begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix} \times (-2)\begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = -8\begin{pmatrix} 1 & 9 \\ 0 & -8 \end{pmatrix}$$

$$[\text{adj}(2A)]^n = (-2)^n \begin{bmatrix} 1 & (-1)^n[2^n - (-1)^n] \\ 0 & (-1)^n 2^n \end{bmatrix}$$

$$[\text{adj}(2A)]^{10} = 2^{10}\begin{pmatrix} 1 & -(2^{10}-1) \\ 0 & 2^{10} \end{pmatrix}$$

Now, $A^{10} - [\text{adj}(2A)]^{10}$

$$= \begin{bmatrix} 2^{10} & 2^{10}-1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2^{10} & -(2^{20}-2^{10}) \\ 0 & 2^{20} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2\cdot 2^{10} - 2^{20} - 1 \\ 0 & 1 - 2^{20} \end{bmatrix}$$

$\det | A^{10} - \text{adj}(2A)^{10} | = 0$

$\therefore \det(A^4) + \det[A^{10} - \text{adj}(2A)^{10}] = (16)^4 + 0 = 16$

Question117

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + ... + A^{20}$, then the sum of all the elements of the matrix M is equal to
[2021, 27 July Shift-II]

Answer: 2020

Solution:

Solution:

We have, $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

.....
.....

$A^n = \begin{bmatrix} 1 & n & \sum n \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$

Now, $M = A + A^2 + A^3 + \ldots + A^{20}$

$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} +$

$\ldots + \begin{bmatrix} 1 & n & \sum n \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 20 & \sum n & \sum(\sum r) \\ 0 & 20 & \sum n \\ 0 & 0 & 20 \end{bmatrix}$

Now, $\sum_{r=1}^{20} n = 1 + 2 + \ldots + 20 = \frac{20 \times 21}{2} = 210$

$\sum_{r=1}^{20} (\sum r) = \sum_{r=1}^{20} \frac{r(r+1)}{2} = \frac{1}{2} \sum_{r=1}^{20} (r^2 + r)$

$= \frac{1}{2}\left[\frac{20 \times 21 \times 41}{6} + \frac{20 \times 21}{2} \right]$

$= \frac{1}{2}[2870 + 210] = 1540$

Hence, $M = \begin{bmatrix} 20 & 210 & 1540 \\ 0 & 20 & 210 \\ 0 & 0 & 20 \end{bmatrix}$

Sum of all elements $= 2020$.

Question118

$$S = \left\{ n \in N \;\middle|\; \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^2 \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\}$$

$$\forall\, a, b, c, d \in R$$

, where $i = \sqrt{-1}$. Then the number of 2-digit numbers in the set S is [2021, 25 July Shift-1]

Answer: 11

Solution:

Solution:

Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $A = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n$

$\Rightarrow AX = 1X$

$A = 1$

$$A^2 = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$$

$$\Rightarrow A = i\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\Rightarrow A^8 = i^4 1 = 1$

$\therefore n = 8$

$\Rightarrow n$ is a multiple of 8 .

16, 24, ...96

Number of elements $= \frac{96-16}{8} + 1 = 11$

Question119

If $P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$, then P^{50} is

[2021, 25 July Shift-II]

Options:

A. $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$

Answer: A

Solution:

Solution:

Given, $P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

$\Rightarrow P^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow P^3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$

$\Rightarrow P^4 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$\vdots$

$\Rightarrow P^n = \begin{bmatrix} 1 & 0 \\ \frac{n}{2} & 1 \end{bmatrix}$

Hence, $P^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

Question120

Let $A = [a_{ij}]$ be a real matrix of order 3×3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for $i = 1, 2, 3$. Then,
the sum of all the entries of the matrix A^3 is equal to
[2021, 22 July Shift-II]

Options:

A. 2

B. 1

C. 3

D. 9

Answer: C

Solution:

Solution:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = A$$

Sum of elements of each row is 1 .

Let X be $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Then, $AX = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$AX = \begin{bmatrix} a_{11}+ & a_{12}+ & a_{13} \\ a_{21}+ & a_{22}+ & a_{23} \\ a_{31}+ & a_{32}+ & a_{33} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$AX = X$
Replace X with AX.
$A \cdot AX = AX \Rightarrow A^2X = X$
Again, replace X with AX,
$A^2(AX) = AX$
$A^3X = X$

Let $A^3 = \begin{bmatrix} a_{11}' & a_{12}' & a_{13}' \\ a_{21}' & a_{22}' & a_{23}' \\ a_{31}' & a_{32}' & a_{33}' \end{bmatrix}$

Then,

$$A^3X = \begin{bmatrix} a_{11}' & a_{12}' & a_{13}' \\ a_{21}' & a_{22}' & a_{23}' \\ a_{31}' & a_{32}' & a_{33}' \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A^3X = \begin{bmatrix} a_{11}' + a_{12}' + a_{13}' \\ a_{21}' + a_{22}' + a_{23}' \\ a_{31}' + a_{32}' + a_{33}' \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

So, $a_{11}' + a_{12}' + a_{13}' = 1$

$a_{21}' + a_{22}' + a_{23}' = 1$

$a_{31}' + a_{32}' + a_{33}' = 1$

$\therefore$ Sum $= 3$

Question121

Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then, the number of 3×3 matrices B with entries from the set {1, 2, 3, 4, 5} and satisfying AB = BA is
[2021, 22 July Shift-II]

Answer: 3125

Solution:

Solution:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} B = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & a_1 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

$AB = BA$

$$\begin{pmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{pmatrix} = \begin{pmatrix} a_2 & a_1 & a_3 \\ b_2 & b1 & b_3 \\ c_2 & c1 & c_3 \end{pmatrix}$$

$$\begin{cases} b_1 = a_2 \\ b_2 = a_1 \\ b_3 = a_3. \end{cases} \quad \begin{cases} a_1 = b_2 \\ a_2 = b_1 \\ a_3 = b_3. \end{cases} \quad \begin{cases} C_1 = C_2 \\ C_2 = C_1 \\ C_3 = C_3. \end{cases}$$

$$B = \begin{pmatrix} a_1 & a_2 & a_3 \\ a_2 & a_1 & a_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

Number of distinct elements in B is 5 $\{a_1, a_2, a_3, c_1, c_3\}$ and according to question, $a_{ij} \in \{1, 2, 3, 4, 5\}$.
So, number of matrices $= 5 \times 5 \times 5 \times 5 \times 5 = 3125$

Question122

Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order 3×3. If $B = [b_{ij}]$, then b_{13} is equal to ...
[2021, 20 July Shift-1]

Answer: 910

Solution:

Solution:

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$B = 7A^{20} - 20A^7 + 2I$

$$\therefore A^2 = A \cdot A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^2 = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^4 = A^3 A = \begin{pmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^4 = \begin{pmatrix} 1 & -4 & 6 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^n = \begin{pmatrix} 1 & -n & \frac{n^2 - n}{2} \\ 0 & 1 & -n \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore \ b_{13} = 7\left(\frac{20 \times 19}{2}\right) - 20\left(\frac{7 \times 6}{2}\right) + 2(0)$$

$$\Rightarrow b_{13} = 1330 - 420 = 910$$

Question123

If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$, **and** $Q = A^T BA$, **then the inverse of the matrix** $AQ^{2021}A^T$ **is equal to**
[26 Aug 2021 Shift 1]

Options:

A. $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$

B. $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

D. $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

Answer: B

Solution:

Solution:

$$AA^T = \begin{vmatrix} \frac{1}{5} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{vmatrix} \begin{vmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{vmatrix}$$

$$AA^T \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

Now, $Q^2 = A^TBAA^TBA$

$\Rightarrow Q^2 = A^TB^2A$

Again, $Q^3 = (A^TBA)(A^TB^2A) = A^TB^3A$

Similarly,

$Q^{2021} = A^TB^{2021}A$

$AQ^{2021}A^T = A(A^TB^{2021}A)A^T$

$= (AA^T)B^{2021}(AA^T) = B^{2021}$

$$B = \begin{vmatrix} 1 & 0 \\ i & 1 \end{vmatrix}$$

$$B^2 = \begin{vmatrix} 1 & 0 \\ 2i & 1 \end{vmatrix}, \text{similarly } B^{2021} = \begin{vmatrix} 1 & 0 \\ 2021i & 1 \end{vmatrix}$$

$$(B^{2021})^{-1} = \frac{\text{adj}(B^{2021})}{|B^{2021}|} = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$$

Question124

Let A be a 3 × 3 real matrix. If det(2 Adj(2 Adj(Adj(2A)))) = 2^{41} , then the value of det(A^2) equal
[26 Aug 2021 Shift 2]

Answer: 4

Solution:

Solution:
We have, A is a square matrix of 3×3. Now,
$2\ \text{Adj}(2\ \text{Adj}(\text{Adj}(2A)))$
$= 2\ \text{Adj}(2\ \text{Adj}(2^{3-1}\ \text{adj}\ A))$ [$\because \text{adj}(KA) = K^{n-1}\ \text{adj}\ A$]
$= 2\ \text{Adj}(2\ \text{Adj}(4\ \text{Adj}\ A))$
$= 2\ \text{Adj}(2 \times 4^{3-1}\ \text{Adj Adj}\ A)$
$= 2\ \text{Adj}(32\ \text{Adj Adj}\ A)$
$= 2 \times 32^{3-1}\ \text{Adj Adj Adj}\ A$
$= 2^{11}\ \text{Adj Adj Adj}\ A$
$= 2^{11}\ \text{Adj}(|A|^{3-2}A)$
$= 2^{11}\ \text{Adj}(|A|A)$
$= 2^{11} \times |A|^{3-1}\ \text{Adj}\ A$
$= 2^{11} \times |A|^{2}\ \text{Adj}\ A$
Now, $|2\ \text{Adj}(2\ \text{Adj}(\text{Adj}(2A)))| = 2^{41}$
$\Rightarrow |2^{11} \times |A|^2\ \text{Adj}\ A = 2^{41}$
$\Rightarrow \left(2^{11}\right)^3 (|A|^2)^3 |\text{Adj}\ A| = 2^{41}$
$\Rightarrow 2^{33} |A|^6 |A|^{3-1} = 2^{41}$
$\Rightarrow |A|^6 \times |A|^2 = 2^8$
$\Rightarrow |A|^8 = 2^8$
$\Rightarrow |A| = \pm 2$
Now, $|A^2| = |A|^2 = (\pm 2)^2 = 4$

Question125

Let A = $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = \alpha I + \beta A$, $\alpha, \beta \in R$, I is a 2×2 identity matrix, then $4(\alpha - \beta)$ is equal to :
[27 Jul 2021 Shift 1]

Options:

A. 5

B. $\frac{8}{3}$

C. 2

D. 4

Answer: D

Solution:

Solution:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}, |A| = 6$$

$$A^{-1} = \frac{\text{ad j}A}{|A|} = \frac{1}{6}\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -\beta & 4\beta \end{bmatrix}$$

$$\left.\begin{array}{l} \alpha + \beta = \frac{2}{3} \\ \beta = -\frac{1}{6} \end{array}\right\} \Rightarrow \alpha = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}$$

$$4(\alpha - \beta) = 4(1) = 4$$

Question126

Let A and B be two 3 × 3 real matrices such that$(A^2 - B^2)$ is invertible matrix. If $A^5 = B^5$ and$A^3B^2 = A^2B^3$, then the value of the determinant ofthe matrix $A^3 + B^3$ is equal to :
[27 Jul 2021 Shift 2]

Options:

A. 2

B. 4

C. 1

D. 0

Answer: D

Solution:

Solution:
$C = A^2 - B^2; |C| \neq 0$
$A^5 = B^5$ and $A^3B^2 = A^2B^2$
Now, $A^5 - A^3B^2 = B^5 - A^2B^3$
$\Rightarrow A^3(A^2 - B^2) + B^3(A^2 - B^2) = 0$
$\Rightarrow (A^3 + B^3)(A^2 - B^2) = 0$
Post multiplying inverse of $A^2 - B^2$:
$A^3 + B^3 = 0$

Question127

Let $A = \{a_{ij}\}$ be a 3×3 matrix, wherea_{ij} = $\begin{cases} (-1)^{j-1} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$ **then**

$det(3Adj(2A^{-1}))$ is equal to _____.
[20 Jul 2021 Shift 2]

Answer: 108

Solution:

Solution:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$|A| = 4$
$|3adj(2A^{-1})| = |3.2^2 adj(A^{-1})|$
$= 12^3 |adj(A^{-1})| = 12^3 |A^{-1}|^2 = \frac{12^3}{|A|^2} = \frac{12^3}{16} = 108$

Question128

If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$, then the value of K is
[27 Aug 2021 Shift 1]

Options:

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. -1

D. 1

Answer: A

Solution:

Solution:
Given matrix,

$$A = \begin{bmatrix} 0 & 2 \\ K & -1 \end{bmatrix}$$

Characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -\lambda & 2 \\ K & -1 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda(1 + \lambda) - 2k = 0$$

$$\Rightarrow \lambda^2 + \lambda - 2k = 0$$

$\because$ Every square matrix satisfied its own characteristic equation.

$$\therefore A^2 + A - 2KI = 0$$

$$\Rightarrow A^2 = 2KI - A$$

$$\Rightarrow A^4 = 4K^2I + A^2 - 2(2KI)(A)$$

$$\Rightarrow A^4 = 4K^2I + 2KI - A - 4KA$$

$$\Rightarrow A^4 = (4K^2 + 2K)I - (1 + 4K)A \quad ...(i)$$

Now, $A(A^3 + 3I) = 2I$

$$\Rightarrow A^4 = 2I - 3A \quad ...(ii)$$

Comparing Eqs. (i) and (ii), we get

$$1 + 4K = 3$$

$$\Rightarrow K = \frac{1}{2}$$

Question129

Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to

[2021, 26 Aug. Shift-II]

Options:

A. $A^6 - A$

B. A^5

C. $A^5 - A$

D. A^6

Answer: A

Solution:

Solution:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Now,

$$A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

.....
.....

$$A^n = \begin{pmatrix} 1 & 0 & 0 \\ n-2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^{2025} - A^{2020} = \begin{pmatrix} 1 & 0 & 0 \\ 2024 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 2019 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now,

$$A^6 - A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore A^{2025} - A^{2020} = A^6 - A$

Question130

Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix

$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal to:

[Jan. 7, 2020 (I)]

Options:

A. A

B. I_3

C. A^2

D. A^3

Answer: D

Solution:

Solution:
Solution of $x^2 + x + 1 = 0$ is ω, ω^2
So, $\alpha = \omega$ and
$\omega^4 = \omega^3 . \omega = 1 . \omega = \omega$

$$A^2 = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\Rightarrow A^4 = I$

$\Rightarrow A^{30} = A^{28} \times A^3 = A^3$

Question131

The number of all 3×3 matrices A, with enteries from the set $\{-1, 0, 1\}$ such that the sum of the diagonal elements of AA^T is 3, is_____. [NA Jan. 8, 2020 (I)]

Answer: 672

Solution:

Solution:

Let $A = [a_{ij}]_{3 \times 3}$

It is given that sum of diagonal elements of AA^T is 3 i.e., $tr(AA^T) = 3$

$a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + \ldots\ldots + a_{33}^2 = 3$

Possible cases are

$$\left.\begin{array}{rcl} 0,0,0,0,0,0,1,1,1 & \rightarrow & 1 \\ 0,0,0,0,0,0,-1,-1,-1, & \rightarrow & 1 \\ 0,0,0,0,0,0,1,1,-1 & \rightarrow & 3 \\ 0,0,0,0,0,0,-1,1,-1 & \rightarrow & 3 \end{array}\right\} \; {}^9C_6 \times 8 = 84 \times 8 = 672$$

Question132

If $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $10A^{-1}$ is equal to: [Jan. 8, 2020 (II)]

Options:

A. $A - 4I$

B. $6I - A$

C. $A - 6I$

D. $4I - A$

Answer: C

Solution:

Solution:
Characteristics equation of matrix ' A ' is $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 2 \\ 9 & 4-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 6\lambda - 10 = 0$$

$\therefore A^2 - 6A - 10I = 0$
$\Rightarrow A^{-1}(A^2) - 6A^{-1} - 10I\,A^{-1} = 0$
$\Rightarrow 10A^{-1} = A - 6I$

Question133

If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true?
[Sep. 04, 2020 (I)]

Options:

A. $0 \le a^2 + b^2 \le 1$

B. $a^2 - d^2 = 0$

C. $a^2 - c^2 = 1$

D. $a^2 - b^2 = \frac{1}{2}$

Answer: D

Solution:

Solution:

$\because A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$

$\therefore A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}, n \in N$

$\because A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore A^5 = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore a = \cos 5\theta, b = i\sin 5\theta = c, d = \cos 5\theta$

$\therefore a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

$a^2 - c^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

$a^2 - d^2 = \cos^2 5\theta - \cos^2 5\theta = 1$

$a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos 10\theta = \cos\frac{10\pi}{24}$

and $0 < \cos\frac{5\pi}{12} < 1 \Rightarrow 0 \le a^2 + b^2 \le 1$

$\therefore a^2 - b^2 = \frac{1}{2}$ is wrong.

Question134

Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to _____.

[NA Sep. 03, 2020 (I)]

Answer: 10

Solution:

Solution:

$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$x^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix} \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$= \begin{bmatrix} (x^2+1)^2 + x^2 & x(x^2+1) + x \\ x(x^2+1) + x & x^2+1 \end{bmatrix}$

Given that $(x^2+1)^2+x^2=109$
$x^4+3x^2-108=0$
$\Rightarrow (x^2+12)(x^2-9)=0$
$\therefore x^2=9$
$a_{22}=x^2+1=9+1=10$

Question135

Let a, b, c ∈ R be all non-zero and satisfy$a^3+b^3+c^3=2$. If the matrix

$A=\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ satisfies$A^TA=I$, then a value of abc can be:

[Sep. 02, 2020 (II)]

Options:

A. $-\frac{1}{3}$

B. $\frac{1}{3}$

C. 3

D. $\frac{2}{3}$

Answer: B

Solution:

Solution:
Given : $A^TA=I$

$$\Rightarrow = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \sum a^2 & \sum ab & \sum ab \\ \sum ab & \sum a^2 & \sum ab \\ \sum ab & \sum ab & \sum a^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So, $\sum a^2=1$ and $\sum ab=0$
Now, $a^3+b^3+c^3-3abc$
$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= (a+b+c)(1-0)$

$= \sqrt{(a+b+c)^2} = \sqrt{\sum a^2 + 2\sum ab} = \pm 1$

$\Rightarrow 2 - 3abc = 1 \Rightarrow abc = \frac{1}{3}$

or $2 - 3abc = -1 \Rightarrow abc = 1$.

Question136

Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then $|p|$ is :

[Jan. 11, 2019 (I)]

Options:

A. $\frac{1}{\sqrt{5}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{\sqrt{6}}$

Answer: C

Solution:

Solution:

$$A = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}$$

$$\therefore A \cdot A^T = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix} \times \begin{bmatrix} 0 & p & p \\ 2q & q & -q \\ r & -r & r \end{bmatrix}$$

$$= \begin{bmatrix} 4q^2+r^2 & 2q^2-r^2 & -2q^2+r^2 \\ 2q^2-r^2 & p^2+q^2+r^2 & p^2-q^2-r^2 \\ -2q^2+r^2 & p^2-q^2-r^2 & p^2+q^2+r^2 \end{bmatrix}$$

Given, $AA^T = I$

$\therefore 4q^2 + r^2 = p^2 + q^2 + r^2 = 1$

$\Rightarrow p^2 - 3q^2 = 0$ and $r^2 = 1 - 4q^2$

and $2q^2 - r^2 = 0 \Rightarrow r^2 = 2q^2$

$\therefore p^2 = \frac{1}{2}$, $q^2 = \frac{1}{6}$ and $r^2 = \frac{1}{3}$

$\therefore |p| = \frac{1}{\sqrt{2}}$

Question137

Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that $Q - P^5 = I_3$. Then $\frac{q_{21} + q_{31}}{q_{32}}$ is equal to:

[Jan. 12, 2019 (I)]

Options:

A. 10

B. 135

C. 15

D. 9

Answer: A

Solution:

Solution:

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix}$$

$$\Rightarrow P^4 = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 90 & 12 & 1 \end{bmatrix}$$

$$\Rightarrow P^5 = = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 90 & 12 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 135 & 15 & 1 \end{bmatrix}$$

$\because Q - P^5 = I_3$

$\therefore Q = I_3 + P^6 = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$

$\frac{q_{21} + q_{31}}{q_{32}} = \frac{15 + 135}{15} = 10$

Question138

Let $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, ($\alpha \in R$) such that $A^{32} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ Then a value of α is :
[April 8, 2019 (I)]

Options:

A. $\frac{\pi}{32}$

B. 0

C. $\frac{\pi}{64}$

D. $\frac{\pi}{16}$

Answer: C

Solution:

Solution:

$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$

Similarly, $A^4 = A^2 . A^2 = \begin{bmatrix} \cos 4\alpha & -\sin 4\alpha \\ \sin 4\alpha & \cos 4\alpha \end{bmatrix}$

and so on $A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Then $\sin 32\alpha = 1$ and $\cos 32\alpha = 0$

$\Rightarrow 32\alpha = n\pi + (-1)^n \frac{\pi}{2}$ and $32\alpha = 2n\pi + \frac{\pi}{2}$

$\Rightarrow \alpha = \frac{n\pi}{32} + (-1)^n \frac{\pi}{64}$ and $\alpha = \frac{n\pi}{16} + \frac{\pi}{64}$ where $n \in z$

Put $n = 0$, $\alpha = \frac{\pi}{64}$

Hence, required value of α is $\frac{\pi}{64}$

Question139

The total number of matrices $A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$, (x, y∈R, x ≠ y) for which $A^T A = 3I_3$ is:
[April 09, 2019 (II)]

Options:

A. 2

B. 3

C. 6

D. 4

Answer: D

Solution:

Solution:
Given, $A^T A = 3I_3$

$$\begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix} = 3I$$

$$\Rightarrow \begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\Rightarrow 8x^2 = 3$ and $6y^2 = 3 \Rightarrow x = \pm \sqrt{\frac{3}{8}}$ and $y = \pm \sqrt{\frac{1}{2}}$

Number of combinations of $(x, y) = 2 \times 2 = 4$

Question140

If A is a symmetric matrix and B is a skew-symmetrix matrix such that $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to:

[April 12, 2019 (I)]

Options:

A. $\begin{bmatrix} -4 & -1 \\ -1 & 4 \end{bmatrix}$

B. $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

C. $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

D. $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$

Answer: B

Solution:

Solution:

Let $A = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$ and $B = \begin{bmatrix} 0 & d \\ -d & 0 \end{bmatrix}$

Then, $A + B = \begin{bmatrix} a & c+d \\ c-d & b \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$

On comparing each term,

$a = 2, b = -1, c - d = 5, c + d = 3$

$\Rightarrow a = 2, b = -1, c = 4, d = -1$

Now, $AB = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

Question141

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is?
[Online April 16, 2018]

Options:

A. 211

B. 210

C. 231

D. 251

Answer: C

Solution:

Solution:

Here $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

also $A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix}$

and, $A^4 = A^3 \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 10 & 4 & 1 \end{bmatrix}$$

On observing the pattern, we come to a conclusion that,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ n & 1 & 0 \\ \frac{n(n+1)}{2} & n & 1 \end{bmatrix}$$

$\therefore A^{20} = [1, 0, 0; 20, 1, 0; 210, 20, 1]$

Therefore, sum of first column of $A^{20} = [1 + 20 + 210] = 231$

Question142

For two 3 × 3 matrices A and B, let $A + B = 2B^T$ and $3A + 2B = I_3$, where B^T is the transpose of B and I_3 is 3 × 3 identity matrix. Then : quad

[Online April 9, 2017]

Options:

A. $5A + 10B = 2I_3$

B. $10A + 5B = 3I_3$

C. $B + 2A = I_3$

D. $3A + 6B = 2I_3$

Answer: B

Solution:

Solution:

$A^T + B^T = 2B$

$\because [(A + B)^T = (2B^T)^T]$

$$\Rightarrow B = \frac{A^T + B^T}{2} = A + \left(\frac{B^T + A^T}{2}\right) = 2B^T$$

$$\Rightarrow 2A + A^T = 3B^T \Rightarrow A = \frac{3B^T - A^T}{2}$$

Also, $3A + 2B = I_3$ .....(i)

$$\Rightarrow 3(3B^T - A^T 2) + 2\left(\frac{A^T + B^T}{2}\right) = I_3$$

$\Rightarrow 11B^T - A^T = 2I_3$ .....(ii)

Add (i) and (ii)

$35 \sim B = 7I_3$

$\Rightarrow B = \frac{I_3}{5} \Rightarrow 11\frac{I_3}{5} - A = 2I_3$

$\Rightarrow 11\frac{I_3}{5} - 2I_3 = A \Rightarrow A = \frac{I_3}{5}$

$\because 5A = 5B = I_3$

$\Rightarrow 10A + 5B = 3I_3$

Question143

If P = $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and Q = PAP^T, then P^T Q^{2015}P is ;

[Online April 9, 2016]

Options:

A. $\begin{bmatrix} 0 & 2015 \\ 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$

Answer: C

Solution:

Solution:

$$P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \quad P^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$PP^T = P^T P = I$

$Q^{2015} = (PAP^T)(PAP^T)$————(2015 terms)

$= PA^{2015}P^T$

$P^T Q^{2015} P = A^{2015}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{2015} = \begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$$

Question144

If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then which one of the following statements is not correct?
[Online April 10, 2015]

Options:

A. $A^2 + I = A(A^2 - I)$

B. $A^4 - I = A^2 + I$

C. $A^3 + I = A(A^3 - I)$

D. $A^3 - I = A(A - I)$

Answer: A

Solution:

Solution:
Given that

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow A^2 = -I$$

$$A^3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$A^2 + I = A^3 - A$
$-I + I = A^3 - A$
$A^3 \neq A$

Question145

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to: [2015]

Options:

A. (2,1)

B. (-2,-1)

C. (2,-1)

D. (-2,1)

Answer: B

Solution:

Solution:
Given that $AA^T = 9I$

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -1 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+4+4 & 2+2-4 & a+4+2b \\ 2+2-4 & 4+1+4 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$\Rightarrow a + 4 + 2b = 0 \Rightarrow a + 2b = -4$ .....(i)
$2a + 2 - 2b = 0 \Rightarrow 2a - 2b = -2$ .....(ii)
$\Rightarrow a - b = -1$
Subtract (ii) from (i)
$a + 2b = -4$
$a - b = -1$
$-\quad +\quad +$

$3b = -3$
$b = -1$
and $a = -2$
(a, b) = (–2, –1)

Question146

If A = $\begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ be such that AB = $\begin{bmatrix} 6 \\ 8 \end{bmatrix}$,then:

[Online April 12, 2014]

Options:

A. $y = 2x$

B. $y = -2x$

C. $y = x$

D. $y = -x$

Answer: A

Solution:

Solution:

Let $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} y + 2x + x \\ 3y - x + 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} y+3x \\ 3y-x+2 \end{bmatrix}$

$\Rightarrow y+3x=6$ and $3y-x=6$

On solving, we get

$x=\frac{6}{5}$ and $y=\frac{12}{5}$

$\Rightarrow y=2x$

Question147

Let A and B be any two 3 × 3 matrices. If A is symmetric and B is skewsymmetric, then the matrix AB – BA is: [Online April 19, 2014]

Options:

A. skewsymmetric

B. symmetric

C. neither symmetric nor skewsymmetric

D. I or – I, where I is an identity matrix

Answer: B

Solution:

Solution:

Let A be symmetric matrix and B be skew symmetric matrix.

$\therefore A^T = A$ and $B^T = -B$

Consider $(AB-BA)^T = (AB)^T - (BA)^T$

$= B^T A^T - A^T B^T = (-B)(A) - (A)(-B)$

$= -BA + AB = AB - BA$

This shows $AB - BA$ is symmetric matrix.

Question148

If p, q, r are 3 real numbers satisfying the matrix equation,

$$\begin{bmatrix} p & q & r \end{bmatrix} \begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 \end{bmatrix}$$ then $2p+q-r$ equals :

[Online April 22, 2013]

Options:

A. – 3

B. – 1

C. 4

D. 2

Answer: A

Solution:

Solution:
Given

$$[\ p \quad q \quad r\ ]\begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = [\ 3 \quad 0 \quad 1\ ] \Rightarrow [\ 3p+3q+2r \quad 4p+2q \quad p+3q+2r\ ] = [\ 3 \quad 0 \quad 1\ ]$$

$\Rightarrow 3p+3q+2r=3$ .....(i)
$4p+2q=0 \Rightarrow q=-2p$ .....(ii)
$p+3q+2r=1$ .....(iii)
On solving (i), (ii) and (iii), we get
$p=1, q=-2, r=3$
$\therefore 2p+q-r=2(1)+(-2)-(3)=-3.$

Question149

The matrix $A^2+4A-5I$, where I is identity matrix and $A=\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, equals
[Online April 9, 2013]

Options:

A. $4\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

B. $4\begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$

C. $32\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

D. $32\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Answer: A

Solution:

Solution:

$$A^2 + 4A - 5I = A \times A + 4A - 5I$$

$$= \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + 4\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9+4-5 & -4+8-0 \\ -8+16-0 & 17-12-5 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix} = 4\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$$

Question150

If $A = \begin{pmatrix} \alpha - 1 \\ 0 \\ 0 \end{pmatrix}$, $B = \begin{pmatrix} \alpha + 1 \\ 0 \\ 0 \end{pmatrix}$ be two matrices, then AB^T is anon-zero matrix for |a| not equal to
[Online May 7, 2012]

Options:

A. 2

B. 0

C. 1

D. 3

Answer: C

Solution:

Solution:

Let $A = \begin{pmatrix} \alpha - 1 \\ 0 \\ 0 \end{pmatrix}, B = \begin{pmatrix} \alpha + 1 \\ 0 \\ 0 \end{pmatrix}$

be two matrices.

$$AB^T = \begin{pmatrix} \alpha - 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \alpha + 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \alpha^2 - 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Thus, AB^T is non-zero matrix for $|\alpha| \neq 1$

Question151

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 7 & -2 & 1 \end{bmatrix}$ then AB equals

[Online May 26, 2012]

Options:

A. I

B. A

C. B

D. 0

Answer: A

Solution:

Solution:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 7 & -2 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Question152

If $\omega \neq 1$ is the complex cube root of unity and matrix $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to
[2011RS]

Options:

A. 0

B. $-H$

C. H^2

D. H

Answer: D

Solution:

Solution:

$$H^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$$

We observed that $H^k = \begin{bmatrix} \omega^k & 0 \\ 0 & \omega^k \end{bmatrix}$

$$\therefore H^{70} = \begin{bmatrix} \omega^{70} & 0 \\ 0 & \omega^{70} \end{bmatrix} = \begin{bmatrix} \omega^{69}\omega & 0 \\ 0 & \omega^{69}\omega \end{bmatrix} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = H \ [\because \omega^{3n} = 1]$$

Question153

Let A and B be two symmetric matrices of order 3.
Statement-1: A(BA) and (AB)A are symmetric matrices.
Statement-2: AB is symmetric matrix if matrix multiplication of A with B is commutative.
[2011]

Options:

A. Statement-1 is true, Statement-2 is true; Statement-2 isnot a correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is false.

C. Statement-1 is false, Statement-2 is true.

D. Statement-1 is true, Statement-2 is true; Statement-2 isa correct explanation for Statement-1.

Answer: A

Solution:

Solution:
Given that A and B are symmetric matrix
A' = A
B' = B
Now (A(BA))' = (BA)'A' = (A'B')A' = (AB)A = A(BA)(∵ product of matrices are associative)
Similarly, ((AB)A)' = A'(B'A') = A (BA) = (AB)A
So, A(BA) and (AB)A are symmetric matrices.
Again (AB)' = B'A' = BA
Now if BA = AB, then AB is symmetric matrix

Question154

The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is
[2010]

Options:

A. 5

B. 6

C. at least 7

D. less than 4

Answer: C

Solution:

Solution:

$\begin{bmatrix} 1 & \ldots & \ldots \\ \ldots & 1 & \ldots \\ \ldots & \ldots & 1 \end{bmatrix}$ are 6 non-singular matrices because 6 blanks will be filled by 5 zeros and 1 one.

$\begin{bmatrix} 1 & \ldots & \ldots \\ \ldots & 1 & \ldots \\ \ldots & \ldots & 1 \end{bmatrix}$ are 6 non-singular matrices.

Total = 6 + 6 = 12
So, required cases are more than 7, non-singular 3 × 3 matrices.

Question155

Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in N$. Then

[2006]

Options:

A. there cannot exist any B such that AB = BA

B. there exist more than one but finite number of B's such that AB = BA

C. there exists exactly one B such that AB = BA

D. there exist infinitely many B¢s such that AB = BA

Answer: D

Solution:

Solution:

Given that $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

Hence, AB = BA only when a = b
∴ There can be infinitely many B's for which AB = BA

Question156

If A and B are square matrices of size n × n such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true?
[2006]

Options:

A. A = B

B. AB = BA

C. either of A or B is a zero matrix

D. either of A or B is identity matrix

Answer: B

Solution:

Solution:
Given that $A^2 - B^2 = (A - B)(A + B)$
$A^2 - B^2 = A^2 + AB - BA - B^2$
$\Rightarrow AB = BA$

Question157

If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all n ≥ 1, by the principle of mathematical induction
[2005]

Options:

A. $A^n = nA - (n - 1)I$

B. $A^n = 2^{n-1}A - (n - 1)I$

C. $A^n = nA + (n - 1)I$

D. $A^n = 2^{n-1}A + (n - 1)I$

Answer: A

Solution:

Solution:

Given that $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $A^3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

Therefore we observed that $A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$

Now $nA-(n-1)I = \begin{bmatrix} n & 0 \\ n & n \end{bmatrix} - \begin{bmatrix} n-1 & 0 \\ 0 & n-1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} = A^n$

$\therefore nA-(n-1)I = A^n$

Question158

If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

[2003]

Options:

A. $\alpha = 2ab, \beta = a^2 + b^2$

B. $\alpha = a^2 + b^2, \beta = ab$

C. $\alpha = a^2 + b^2, \beta = 2ab$

D. $\alpha = a^2 + b^2, \beta = a^2 - b^2$.

Answer: C

Solution:

Solution:

$A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \Rightarrow A \cdot A = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{bmatrix}$

$\alpha = a^2 + b^2; \beta = 2ab$